1. Let $f(x) = -\frac{3}{2}x + 4$

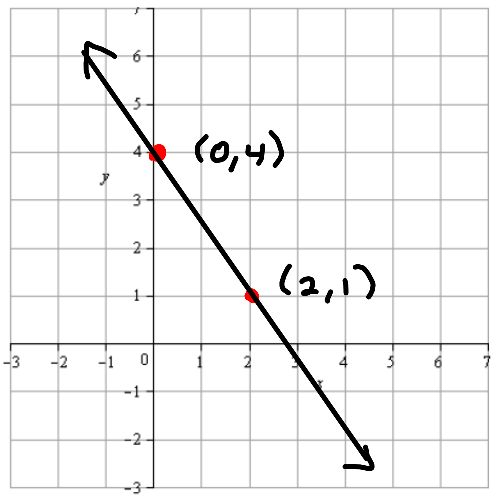
   a. (5 pts) Determine the slope and y-intercept of f.

   $m = -\frac{3}{2}$, $(0,b) = (0,4)$

   b. (5 pts) Use the slope and y-intercept to graph f, in the space on the right.

   c. (5 pts) What's the average rate of change of f?

   $m = -\frac{3}{2}$



2. Suppose y varies jointly as the square root of x and the square of z and inversely as the square root of w.

   a. (5 pts) Write an equation representing the relationship.

   $$y = k\frac{\sqrt{x}\, z^2}{\sqrt{w}}$$

   b. (5 pts) Suppose $y = 24$ when $x = 2$, $z = 9$ and $w = 4$. What, then, is y when $x = 2$, $z = 3$ and $w = 4$?

   Find k

   use k

   $$24 = k\left(\frac{\sqrt{2}\cdot 9^2}{\sqrt{4}}\right) = \frac{81\sqrt{2}}{2}k = 24$$

   $$k = 24\cdot\frac{2}{81\sqrt{2}} = 8\cdot\frac{2}{27\sqrt{2}} = \boxed{\frac{16}{27\sqrt{2}} = k}$$

   $$y = \left(\frac{16}{27\cancel{\sqrt{2}}}\right)\frac{\cancel{\sqrt{2}}\cdot 3^2}{\sqrt{4}}$$

   $$= \frac{16\cdot 9}{27\cdot 2} = \boxed{\frac{8}{3} = y}$$

3. (5 pts each) Compute the discriminant for each of the following quadratic$_{s,}$ and tell me the nature of solutions, specifically, how many distinct solutions there are and whether they're real or non-real. *Do not solve the equations.*

a. $9x^2 - 12x + 4 = 0$

$a=9, b=-12, c=4 \Longrightarrow$

$b^2-4ac = (-12)^2 - 4(9)(4)$

$= 144 - 144$

$= 0 \Longrightarrow$

One (repeated) root

Real.

b. $9x^2 - 18x + 8$

$a=9, b=-18, c=8$

$b^2-4ac = (-18)^2 - 4(9)(8)$

$= 324 - 288 > 0$

$= 36 > 0$

Two real solutions (Rational)

$$\begin{array}{r} 360 \\ -\ 72 \\ \hline 288 \end{array}$$

4. (10 pts) Solve $15x^2 - 92x + 140 = 0$ by any method, but *show all work*!!! Give your final answer in fractional form (lowest terms).

$a=15, b=-92, c=140$

$b^2-4ac = -92^2 - 4(15)(140)$

$= 8464 - 8400$

$= 64$

$\rightsquigarrow \sqrt{64} = 8$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{92 \pm 8}{2(15)} = \frac{100}{30} = \frac{10}{3}$$

$$\frac{84}{30} = \frac{42}{15} = \frac{14}{5}$$

$x \in \left\{\frac{10}{3}, \frac{14}{5}\right\}$

5. (5 pts) Solve $x^2+8x-11=0$ by completing the square.

$x^2+8x = 11$

$x^2+8x+4^2 = 11+16$

$(x+4)^2 = 27$

$\sqrt{(x+4)^2} = \sqrt{27}$

$|x+4| = 3\sqrt{3}$

$x+4 = \pm 3\sqrt{3}$

$\boxed{x = -4 \pm 3\sqrt{3}}$

$3\,\underline{|27}$
$3\,\underline{|9}$
3

6. Complete the square and re-write each of the following in the form $f(x)=a(x-h)^2+k$.

a. (5 pts) $f(x)=x^2+8x-11$

$f(x) = x^2+8x-11$

$\Rightarrow f(x)+11 = x^2+8x$

$\Rightarrow f(x)+11+16 = x^2+8x+4^2$

$\Rightarrow f(x)+27 = (x+4)^2$

$\boxed{f(x) = (x+4)^2-27}$

$x^2+8x-11$

$= x^2+8x+4^2-16-11$

$= (x+4)^2-27$

$\frac{239}{196}\cdot\frac{7}{1} = \frac{239}{28}$

b. (5 pts) $f(x)=7x^2-3x-10$

$f(x) = 7x^2-3x-10$

$f(x) = 7\left(x^2-\frac{3}{7}x-\frac{10}{7}\right)$

$\frac{1}{7}f(x) = x^2-\frac{3}{7}x-\frac{10}{7}$

$\frac{1}{7}f(x)+\frac{10}{7} = x^2-\frac{3}{7}x$

$\frac{1}{7}f(x)+\frac{10}{7}+\frac{9}{196} = x^2-\frac{3}{7}x+\left(\frac{3}{14}\right)^2$

$\frac{1}{7}f(x)+\frac{239}{196} = \left(x-\frac{3}{14}\right)^2$

$\frac{1}{7}f(x) = \left(x-\frac{3}{14}\right)^2-\frac{239}{196}$

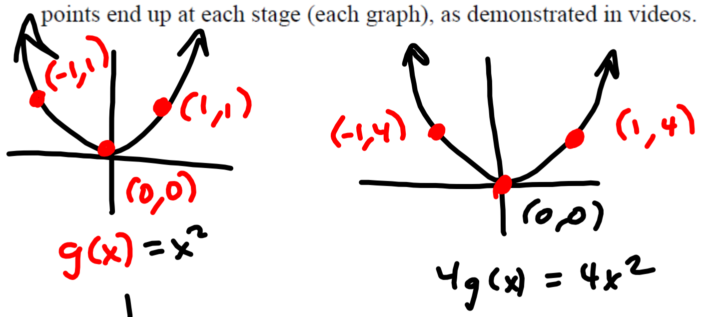
$f(x) = 7\left(x-\frac{3}{14}\right)^2-\frac{239}{28}$

$2\,\underline{|196}$
$2\,\underline{|98}$
$7\,\underline{|49}$
7

$\frac{10}{7}\cdot\frac{28}{28}+\frac{9}{196}$

$\frac{280+9}{196} = \frac{239}{196}$

7. (10 pts) Sketch the graph of $f(x) = 4(x-5)^2 - 3$. I want to see it step-by-step, using techniques I gave you. Your first graph of the basic function should include the points $(-1,1)$, $(0,0)$, and $(1,1)$. Track where those points end up at each stage (each graph), as demonstrated in videos.

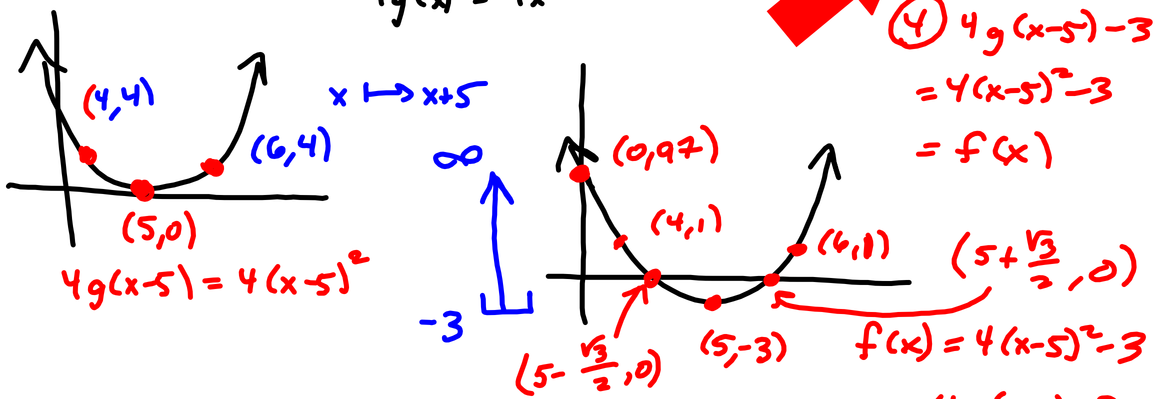


① $g(x) = x^2$

② $4g(x) = 4x^2$

③ $4(g(x-5)) = 4(x-5)^2$

④ $4g(x-5) - 3 = 4(x-5)^2 - 3 = f(x)$



$= 4g(x-5) - 3$

8. (5 pts) Based on your work on the previous, state the domain and range of $f(x)$.

$\mathcal{D} = \mathbb{R} = (-\infty, \infty)$, $\mathcal{R} = [-3, \infty)$

9. (5 pts) Find the x- and y-intercepts for $f(x)$. Indicate and label these points on the graph, above.

$$f(0) = 4(0-5)^2 - 3 = 4(-5)^2 - 3 = 4(25) - 3 = 97 \rightsquigarrow (0,97)$$

$$4(x-5)^2 - 3 = 0$$
$$4(x-5)^2 = 3$$
$$(x-5)^2 = \frac{3}{4}$$
$$x - 5 = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2}$$
$$x = 5 \pm \frac{\sqrt{3}}{2}$$

10. (10 pts) Solve the inequality $3x^2+2x-1>x^2+3x+2$. Give your final answer in set notation and interval notation.

$2x^2$ ...

$\underline{-x^2-3x-2 = -x^2-3x-2}$

$2x^2-x-3>0$

$(2x-3)(x+1)>0$

critical:

$2x-3=0$ $\quad$ $x+1=0$

$2x=3$ $\quad$ $x=-1$

$x=\frac{3}{2}$

Want >0

$+$ $\quad$ $-$ $\quad$ $+$

Yes $\quad$ NO $\quad$ Yes

-1 $\quad$ $\frac{3}{2}$

$x\in(-\infty,-1)\cup(\frac{3}{2},\infty)$

$=\{w \mid w<-1 \text{ OR } w>\frac{3}{2}\}$

$\le$!

Bonus (5 pts) Solve $2x^2-4x+7\le x^2+2x$. Give an *exact* final answer (simplified radical form), in set notation and interval notation. I think this one's easier than #6b!

$x^2-6x+7\le 0$

Solve $x^2-6x+7=0$.

Then analyze the signs!

$x^2-6x \quad =-7$

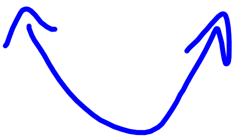
$x^2-6x+3^2=-7+9$

$(x-3)^2=2$

$x-3=\pm\sqrt{2}$

$x=3\pm\sqrt{2}$

Want ≤ 0



$+$ $\quad$ $-$ $\quad$ $+$

No $\quad$ Yes $\quad$ No

$3-\sqrt{2}$ $\quad$ $3+\sqrt{2}$

$x\in[3-\sqrt{2},3+\sqrt{2}]=$

$=\{x \mid 3-\sqrt{2}\le x\le 3+\sqrt{2}\}$

$=\{x \mid 3-\sqrt{2}\le x \text{ AND } x\le 3+\sqrt{2}\}$

$=$ Intersection of $(-\infty,3+\sqrt{2}]$ and $[3-\sqrt{2},\infty)$

No $\quad$ Yes $\quad$ No

$3-\sqrt{2}$ $\quad$ $3+\sqrt{2}$

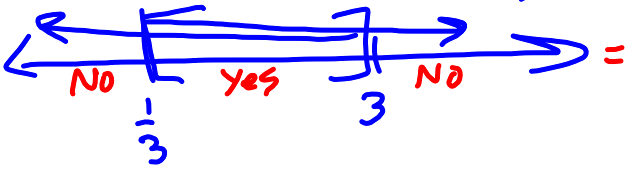
11. Solve the absolute value inequalities. Give your answers in set-builder *and* interval notation.

a. (5 pts) $|3x-5| \le 4$ AND

$-3x-5 \le 4$ AND $3x-5 \ge -4$

$3x \le 9$ $\quad$ $3x \ge 1$

$\{x \mid x \le \frac{9}{3} = 3$ AND $x \ge \frac{1}{3}\} =$



$= [\frac{1}{3}, 3]$

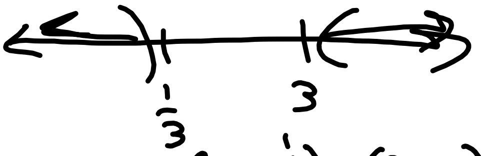
NO

b. (5 pts) $|3x-5| > 4$ OR

$3x-5 > 4$ OR $3x-5 < -4$

$3x > 9$ $\quad$ $3x < 1$

$\{x \mid x > 3$ OR $x < \frac{1}{3}\}$



$x \in (-\infty, \frac{1}{3}) \cup (3, \infty)$

$$\frac{-3x}{-3} \le \frac{9}{-3} \quad \text{FALSE}$$

$$-3x \le 9$$

$$\frac{-3x}{-3} \ge \frac{9}{-3}$$

$$x \ge -3$$

OK

$$-3x \le 9$$

$$x \ge \frac{9}{-3} = -3$$