Calculus I MAT 201 001 Test 3 Ch. 4 F06 Name KEY

Show all your work. No graphing calculators, cell phones, or earphones.

---

1. (20 points) Find the local and absolute extreme values of the function on the given interval: $f(x) = x^3 - 6x^2 + 2$, [-1,2]

$f'(x) = 3x^2 - 12x \underset{=}{\text{SET}} 0$

$3x(x-4) = 0$

$x = 0,\ x = 4 \notin [-1,2]$

$\boxed{f(0) = 2}$ $\boxed{\text{LOCAL MAX ABSOLUTE MAX}}$

$$\begin{array}{r|rrrr} -1 & 1 & -6 & 0 & 2 \\ & & -1 & 7 & -7 \\ \hline & 1 & -7 & 7 & \boxed{-5 = f(-1)} \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 0 & 2 \\ & & 2 & -8 & -16 \\ \hline & 1 & -4 & -8 & \boxed{-14 = f(2)} \end{array}$$

$\boxed{\text{Absolute MIN}}$

---

2. (5 points) Find the limit: $\lim\limits_{x \to \infty} \dfrac{x^3 - 4x + 2}{2x^3 + 5x - 4} = \boxed{\dfrac{1}{2}}$

---

3. (15 points) Given the equation $x^3 - 2 = 0$, use Newton's Method to find x_2 and x_3 to three places if $x_1 = 1$. Show some work.

x_2 = 1.333

x_3 = 1.264

$f'(x) = 3x^2$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = 1 - \dfrac{1^3 - 2}{3(1)^2}$

$= 1 - \dfrac{-1}{3} = 1 + \dfrac{1}{3} = \dfrac{4}{3}$

Using $x_2 = 1.333$

$x_3 \approx 1.333 - \dfrac{(1.333)^3 - 2}{3(1.333)^2}$

$\approx 1.333 - \dfrac{2.368593037 - 2}{5.330667}$

$= 1.333 - \dfrac{.368593037}{5.330667} \approx$

$\approx 1.333 - .069145763$

≈ 1.263854237

USING $x_1 = \frac{4}{3}$:

$\frac{4}{3} - \dfrac{\left(\frac{4}{3}\right)^3 - 2}{3\left(\frac{4}{3}\right)^2} = \frac{4}{3} - \dfrac{\frac{10}{27}}{\frac{16}{3}} =$

$= \frac{4}{3} - \frac{10}{27} \cdot \frac{3}{16}$

$= \frac{4}{3} - \frac{5}{9} \cdot \frac{1}{8}$

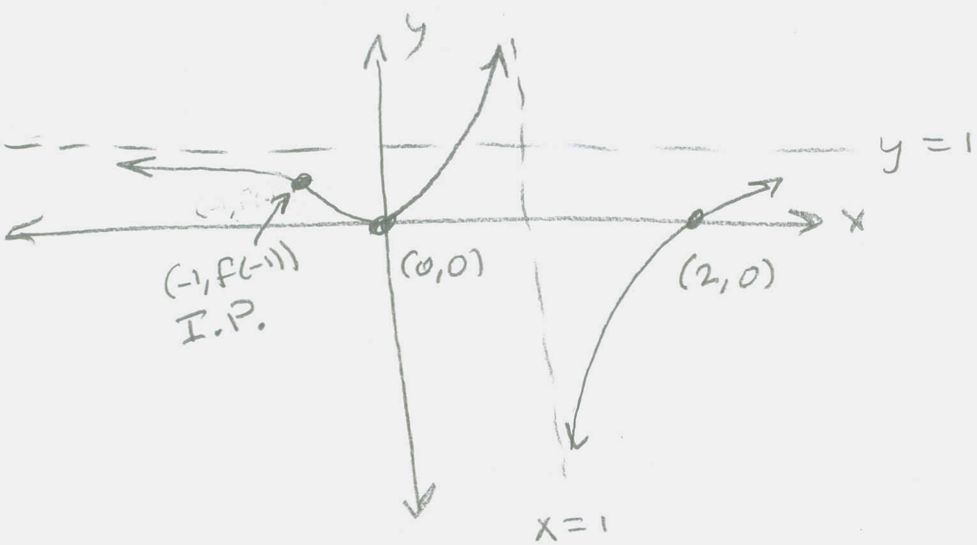
$= \frac{91}{72} \approx 1.264$

4. (20 points) Use the given information to sketch the graph of f(x).

$$\lim_{x\to 1^-} f(x) = \infty, \quad \lim_{x\to 1^+} f(x) = -\infty, \quad \lim_{x\to -\infty} f(x) = 1, \quad \lim_{x\to +\infty} f(x) = 1$$

$f(0) = 0$, $f(2) = 0$

		0		1
f'	neg	|	pos	| pos
f"	neg |	pos		| neg
	-1			1



5. (10 points) If $f'(x) = 6x + 4$, $f(0) = 3$, find $f(x)$.

$$f(x) = \frac{6x^2}{2} + 4x + C$$

$$f(0) = C = 3 \Longrightarrow f(x) = \boxed{3x^2 + 4x + 3}$$

6. (15 points) Find all values c that satisfy the Mean Value Theorem: $f(x) = x^2 - 3x + 5$ on the interval [-1,3]

$$\frac{f(b)-f(a)}{b-a} = \frac{f(3)-f(-1)}{3-(-1)} = \frac{(3)^2-3(3)+5-((-1)^2-3(-1)+5)}{3+1}$$

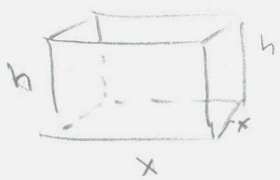
$$= \frac{9-9+5-(1+3+5)}{4} = \frac{5-9}{4} = -\frac{4}{4} = -1$$

$$f'(x) = 2x-3 \overset{\text{SET}}{=} -1 \Longrightarrow$$

$$2x = 2 \Longrightarrow \boxed{x=1}$$

7. (15 points) A metal box has a square base and no top. Its volume is 108 cubic feet. What dimensions require the least amount of metal?

Area = Bottom + 4 sides



$$= x^2 + 4xh$$

$$\text{Volume} = x^2 h = 108$$

$$\Longrightarrow h = \frac{108}{x^2}$$

$$\Longrightarrow \text{Area} = A(x) = x^2 + 4x\left(\frac{108}{x^2}\right)$$

$$= x^2 + \frac{432}{x} =$$

$$= x^2 + 432x^{-1}$$

$$\Longrightarrow A'(x) = 2x - 432x^{-2}$$

$$= \frac{2x^3-432}{x^2} \overset{\text{SET}}{=} 0$$

$$\Longrightarrow x^3 = 216$$

$$\Longrightarrow \boxed{x=6}$$

$x=6 \Longrightarrow$

$$h = \frac{108}{x^2} = \frac{108}{36} = 3$$

$\Longrightarrow$ Box is

$6' \times 6' \times 3'$

$l \times w \times h$

length = l
width = w
height = h
} in feet.