

You may take advantage of up to 15 points of bonus problems on this test. I've sprinkled over 35 bonus points. Unless you write OMIT by the bonus you don't want graded, I will grade the first 15 points'-worth I come to and ignore the rest. Tortoise wins the race.

1. (10 pts) Write the augmented matrix corresponding to the system of linear equations. Take your time and get it right, because #2 depends on your accuracy.

$$\begin{array}{rcl} x + 2y - 5z & = & -13 \\ -3x - 5y + 13z & = & 36 \\ 2x + 6y - 13z & = & -27 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 2 & -5 & -13 \\ -3 & -5 & 13 & 36 \\ 2 & 6 & -13 & -27 \end{array} \right]$$

2. (10 pts) Use your augmented matrix from #1 to solve the system.

$$\begin{array}{l} 3R1 \\ R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 3 & 6 & -15 & -39 \\ -3 & -5 & 13 & 36 \\ 2 & 6 & -13 & -27 \end{array} \right] \xrightarrow{\frac{1}{3}R1} \begin{array}{l} R1+R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -5 & -13 \\ 0 & 1 & -2 & -3 \\ 2 & 6 & -13 & -27 \end{array} \right]$$

$$\begin{array}{l} -2R1 \\ R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} -2 & -4 & 10 & 26 \\ 0 & 1 & -2 & -3 \\ 2 & 6 & -13 & -27 \end{array} \right] \xrightarrow{-\frac{1}{2}R1} \begin{array}{l} R2 \\ R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -5 & -13 \\ 0 & 1 & -2 & -3 \\ 0 & 2 & -3 & -1 \end{array} \right]$$

$$\begin{array}{l} R1 \\ -2R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -5 & -13 \\ 0 & -2 & 4 & 6 \\ 0 & 2 & -3 & -1 \end{array} \right] \xrightarrow{-\frac{1}{2}R2} \begin{array}{l} R1 \\ R2+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -5 & -13 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 5 \end{array} \right]$$

$$\begin{array}{rcl} x + 2y - 5z & = & -13 \\ y - 2z & = & -3 \end{array} \Rightarrow \boxed{z = 5}$$

$$\begin{array}{rcl} y - 2(5) & = & -3 \\ y - 10 & = & -3 \end{array} \Rightarrow \boxed{y = 7}$$

$$\begin{array}{rcl} x + 2(7) - 5(5) & = & -13 \\ x + 14 - 25 & = & -13 \\ x - 11 & = & -13 \end{array} \Rightarrow \boxed{x = -2}$$

$$\{(-2, 7, 5)\}$$

3. (5 pts) Check your answer to #2 by multiplying two matrices together. I'll give you the first one. You can get #2 wrong and still get this one 100% right. Make up 3 numbers if you have to.

$$\begin{bmatrix} 1 & 2 & -5 \\ -3 & -5 & 13 \\ 2 & 6 & -13 \end{bmatrix} \begin{bmatrix} -2 \\ 7 \\ 5 \end{bmatrix} = \begin{bmatrix} -2 + 14 - 25 \\ 6 - 35 + 65 \\ -4 + 42 - 65 \end{bmatrix} = \begin{bmatrix} -13 \\ 36 \\ -27 \end{bmatrix} \checkmark$$

4. (20 pts) For the following word problem, I'm looking for a solid process as much as I'm looking for the final answer. Describing the variables clearly in words and units is worth a lot. Re-writing statements from the problem in your own words and linking them to the equations you derive is worth a lot. A clear solution method and concluding statement is worth a lot.

At the Book Nook, all paperbacks sell for one price and all hardbacks sell for another price. Tanya got six paperbacks and three hardbacks for \$8.25, while Gretta got four paperbacks and five hardbacks for \$9.25. Find the price of hardbacks and paperbacks at the Book Nook.

Let x = the price of a paperback (in \$/paperback)
 y = " " " " " hardback (in \$/hardback)

6 paperbacks & 3 hardbacks are \$8.25

$$6x + 3y = 8.25$$

4 paperbacks and 5 hardbacks are \$9.25

$$4x + 5y = 9.25$$

$$\begin{bmatrix} 6 & 3 & 8.25 \\ 4 & 5 & 9.25 \end{bmatrix} \xrightarrow{-2R1} \begin{bmatrix} -12 & -6 & -16.5 \\ 12 & 15 & 27.75 \end{bmatrix} \xrightarrow{3R2} \begin{bmatrix} -12 & -6 & -16.5 \\ 12 & 15 & 27.75 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{2}R1} \begin{bmatrix} 6 & 3 & 8.25 \\ 0 & 9 & 11.25 \end{bmatrix} \xrightarrow{R1+R2} \begin{bmatrix} 6 & 3 & 8.25 \\ 0 & 9 & 11.25 \end{bmatrix}$$

$$\begin{aligned} 6x + 3y &= 8.25 \\ 9y &= 11.25 \end{aligned}$$

$$\begin{aligned} 6x + 3(1.25) &= 8.25 \\ 6x + 3.75 &= 8.25 \end{aligned}$$

$$6x = 4.5$$

$$x = \frac{4.5}{6} = 0.75$$

$$y = \frac{11.25}{9} = 1.25$$

$$x = \$0.75 \text{ per paperback} \quad y = \$1.25 \text{ per hardback}$$

5. (10 pts) Based on your answer to #4, how much did (Steady) Freddie pay for 7 paperbacks and 11 hardbacks? And if you didn't get an answer for #4, just make up two numbers, stating what they represent, and I'll give full credit if you correctly figure the amount Freddie would have paid at those prices. (, although I *am* set up to grade the *correct* answer to #4 more quickly.)

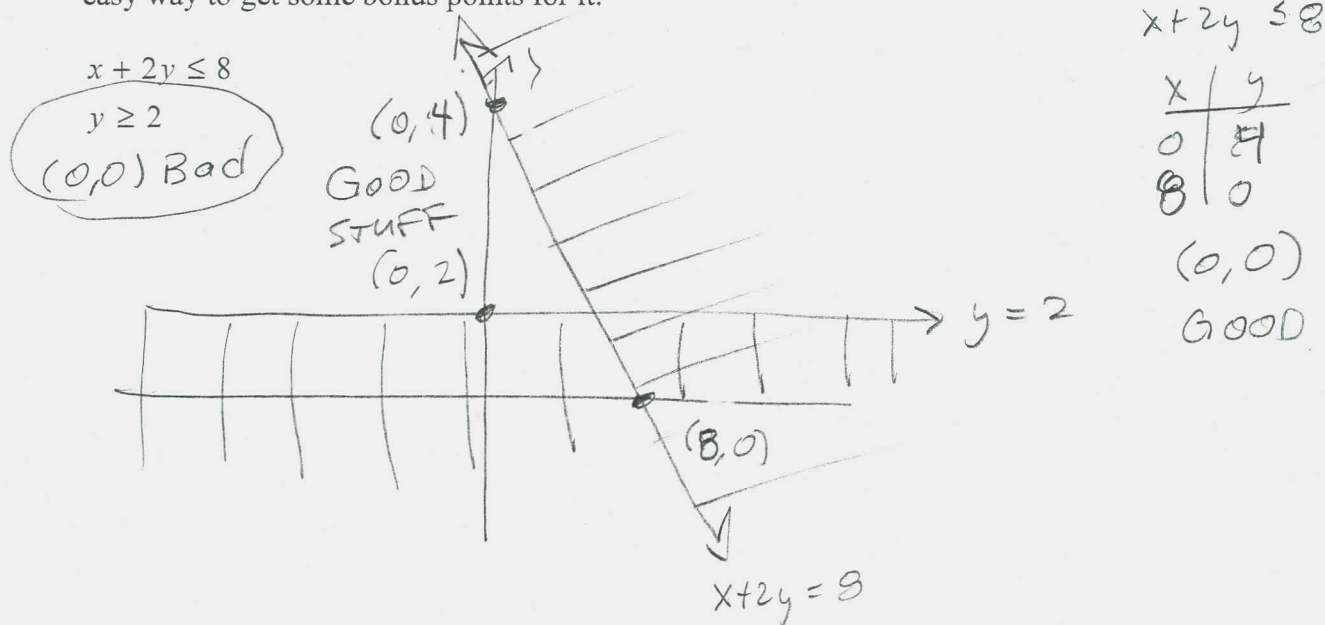
(5 bonus) Use matrix multiplication to answer #5. For this, I'm looking for a 1-by-2 times a 2-by-1, resulting in a 1x1 which holds the answer (in units of dollars). I can see a student getting this right in two different ways.

(5 bonus) Describe the units used in the entries of the matrices you used in the previous bonus problem. Then show how the product of the entries results in the desired units (dollars) by cancelling units.

$$\begin{bmatrix} 7 & 11 \end{bmatrix} \begin{bmatrix} .75 \\ 1.25 \end{bmatrix} = \begin{bmatrix} 7(.75) + 11(1.25) \end{bmatrix} = 19$$

$$\begin{aligned} & (7 \text{ paperbacks}) \left(.75 \frac{\$}{\text{paperback}} \right) + (11 \text{ hardbacks}) \left(1.25 \frac{\$}{\text{hardback}} \right) \\ & = \$19 \end{aligned}$$

(5 pts bonus) Sketch the solution set of the following system of inequalities. However you decide to shade (good stuff or bad stuff), I expect to see you indicate the "good stuff" with the phrase "good stuff." Somehow this skill wasn't covered in lecture, but if you really used your coursecompass as a learning tool, and studied last fall's Test 5, here is an easy way to get some bonus points for it.



6. Let $A = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, $C = [1 \ 2]$, $D = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, and $E = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Evaluate the following, or explain why they cannot be evaluated. In the case one can't be evaluated, your explanation ought to include phrases such as "the number of rows" and "the number of columns."

a. (5 pts) $AD = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0+8 \\ 3-4 \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$

b. (5 pts) AC can't be evaluated. The # of columns in A doesn't match the # of rows in C .

c. (5 pts) $CD = [1 \ 2] \begin{bmatrix} 3 \\ 4 \end{bmatrix} = [3+8] = [11]$

d. (5 pts bonus) $DC = \begin{bmatrix} 3 \\ 4 \end{bmatrix} [1 \ 2] = \begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}$

e. (5 pts bonus) $CB = [1 \ 2] \begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & -1 \end{bmatrix} = [2+2 \quad -3+4 \quad 1-2] = [4 \quad 1 \quad -1]$

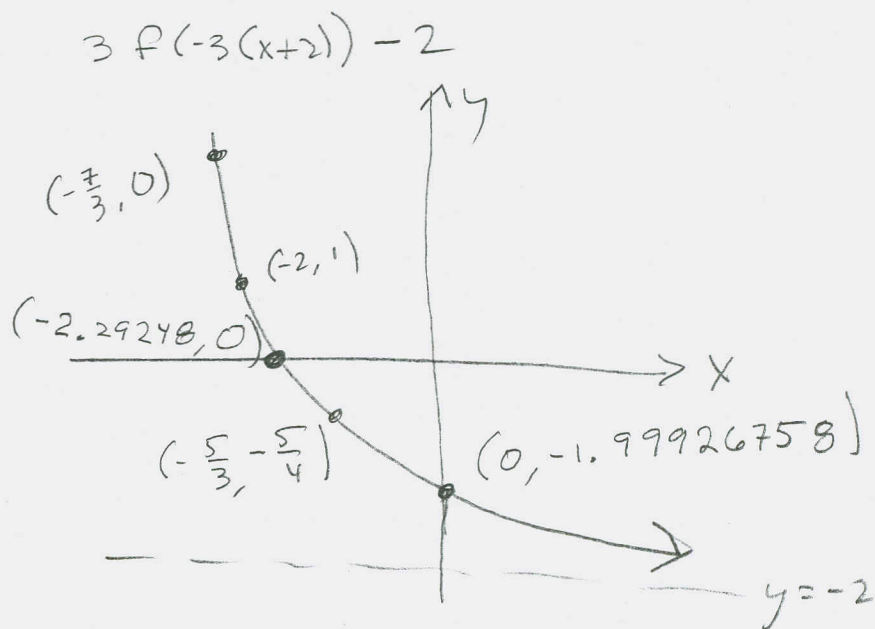
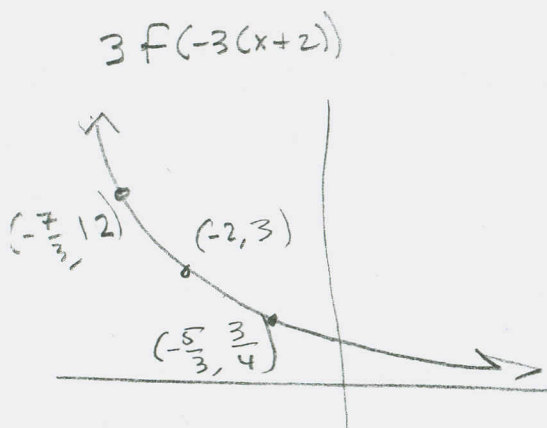
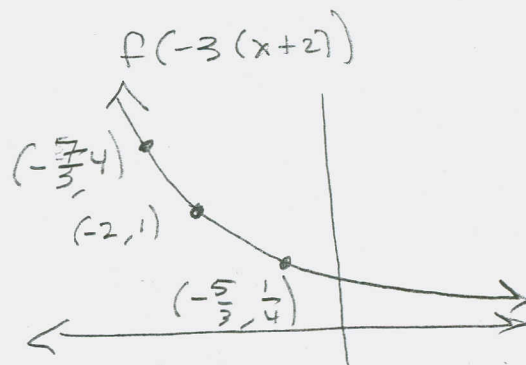
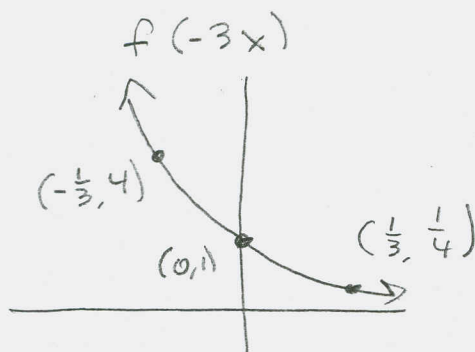
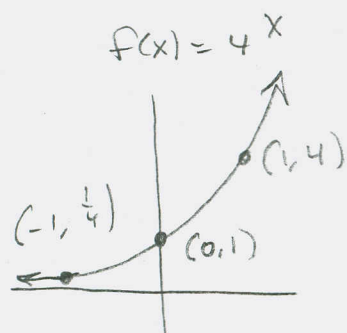
f. (5 pts) $C+E$ can't be evaluated. Matrices are different-sized.

g. (5 pts) $D+E = \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

$$-3x - 6 = -3(x + 2)$$

(10 pts bonus) Sketch the graph of $g(x) = 3 \cdot 4^{-3x-6} - 2$ by transforming the basic function $f(x) = 4^x$, as demonstrated on many occasions in lecture.

(5 pts bonus) Find the x -intercept, accurate to 5 decimal places in the x -coordinate. Label it clearly on your final sketch.



$$3 \cdot 4^{-3x-6} - 2 = 0$$

$$3 \cdot 4^{-3x-6} = 2$$

$$4^{-3x-6} = \frac{2}{3}$$

$$-3x - 6 = \log_4\left(\frac{2}{3}\right)$$

$$-3x = \log_4\left(\frac{2}{3}\right) + 6$$

$$x = -\log_4\left(\frac{2}{3}\right) - 2$$

$$= -\frac{\ln(2/3)}{\ln(4)} - 2$$

$$\approx -2.29248125$$