§1.5

#1 $x^2-2x-63=0$

$x^2-9x+7x-63=0$

$x(x-9)+7(x-9)=0$

$(x-9)(x+7)=0$

$\frac{x\cancel{(x-9)}}{\cancel{x-9}}=x$ $\frac{7\cancel{(x-9)}}{\cancel{x-9}}=7$

$x=-7,9$

$x\in\{-7,9\}$

2 $6x^2-13x+5=0$ $(6)(5)=(2)(3)(5)=30$

$10+3=13$

$(10)(3)=30$

$6x^2-10x-3x+5=0$

$2x(3x-5)-1(3x-5)$

$(3x-5)(2x-1)=0$

$3x-5=0$ ;

$3x=5$ $x=\frac{1}{2}$

$x=\frac{5}{3}$ $\{\frac{1}{2},\frac{5}{3}\}$

FACTORING SLEDGEHAMMER

$6x^2 - 13x + 5 = 0$ $\qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 6, b = -13, c = 5$

$b^2 - 4ac$ = Discriminant

$= (-13)^2 - 4(6)(5) =$

$= 169 - 120$

$= 49$ is a perfect square

It can be factored by ac method.

$$\begin{array}{r} 24 \\ \times 5 \\ \hline 120 \end{array}$$

$= \dfrac{13 \pm \sqrt{49}}{2(6)} = \dfrac{13 \pm 7}{12}$

$\dfrac{13+7}{12} = \dfrac{20}{12} = \dfrac{5}{3}$ $\qquad \dfrac{13-7}{12} = \dfrac{6}{12} = \dfrac{1}{2}$

$x \in \left\{ \frac{1}{2}, \frac{5}{3} \right\}$

$x = \frac{5}{3}$ makes it zero

$\Longrightarrow (x - \frac{5}{3})$ is a factor

$6x^2 - 13x + 5 = 6(x - \frac{5}{3})(x - \frac{1}{2})$

$= (3)(2)(x - 5/3)(x - \frac{1}{2})$

$= 3(x - \frac{5}{3})(2)(x - \frac{1}{2})$

$= (3x - 5)(2x - 1)$

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$x^2 - 7x - 2 = 0$ $\qquad ax^2 + bx + c = 0$

$a = 1, b = -7, c = -2$

$b^2 - 4ac = (-7)^2 - 4(1)(-2)$

$= 49 + 8 = 57$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{7 \pm \sqrt{57}}{2(1)}$

$= \dfrac{7 \pm \sqrt{57}}{2}$

$x \in \left\{ \frac{7 + \sqrt{57}}{2}, \frac{7 - \sqrt{57}}{2} \right\}$

This is how it factors

$x^2 - 7x - 2 = \left(x - \left(\frac{7 + \sqrt{57}}{2}\right)\right)\left(x - \left(\frac{7 - \sqrt{57}}{2}\right)\right)$

4

$b^2 - 26 = 0$

$b^2 = 26$

$b = \pm\sqrt{26}$

Square Root Property

$x^2 = A \Longrightarrow$

$x = \pm\sqrt{A}$

$x^2 = A$

$\sqrt{x^2} = \sqrt{A}$

$|x| = \sqrt{A}$

$\sqrt{x^2} = ?$

$\sqrt{3^2} = \sqrt{9} = 3$

$\sqrt{(-3)^2} = \sqrt{9} = 3$

This is absolute value behavior.

$|x| = \sqrt{A} \Longrightarrow x = \pm\sqrt{A}$

$\underline{x = \sqrt{A}}$ OR $\underline{x = -\sqrt{A}}$

5

$(x+4)^2 = 9$

$u^2 = 9$

$u = \pm 3$

$(x+4) = \pm 3$

$x + 4 = 3 \qquad u + 4 = -3$

$x = -1 \qquad u = -7$

$\{-7, -1\}$

$(x+4)^2 = 9$

$x + 4 = \pm\sqrt{9} = \pm 3$

$x = -4 \pm 3 \to -1, -7$

6. $(x+5)^2 = -64$

$x+5 = \pm\sqrt{-64} = \pm i\sqrt{64}$

$x+5 = \pm i \cdot 8 = \pm 8i$

$x = -5 \pm 8i$

$\sqrt{-1} = i$

$\sqrt{-537} = i\sqrt{537}$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^2 = (a+b)(a+b)$

$= a^2 + ab + ba + b^2$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

7. $z^2 - 24z + \underline{12^2} = (z-12)^2$

$a^2 - 2ab + \underline{b^2}$

$a^2 = z^2 \Longrightarrow a = z$

$-2ab = -\cancel{24}z$

want $b^2$ on this

$-2ab = -24z$

$-2zb = -24z$

$b = \frac{-24z}{-2z} = 12 \rightsquigarrow 12^2 = 144 = b^2$

Divide middle coefficient by 2

square it.

Add.

$z^2 - 24z \underline{+12^2}$

$\frac{24}{2} = 12 \rightsquigarrow 12^2 = 144$

8

$x^2-6x-4=0$ by completing the square

This is an equation.

$x^2-6x+3^2=4+9$

$\frac{6}{2}=3 \rightsquigarrow 3^2=9$

$(x-3)^2=13$

$x-3=\pm\sqrt{13}$

$x=3\pm\sqrt{13}$

$\sqrt{(x-3)^2}=\sqrt{13}$

$|x-3|=\sqrt{13}$

$x-3=\pm\sqrt{13}$

$2x^2+7x-1=0$

This has leading coefficient $\neq 1$. Need to tweak it.

$x^2+\frac{7}{2}x-\frac{1}{2}=0$

$\frac{\frac{7}{2}}{2}=\frac{7}{2}\cdot\frac{1}{2}=\frac{7}{4} \to \left(\frac{7}{2}\right)^2=\frac{49}{4}$

$2 \to 4 \qquad 4 \to 16$

$x^2+\frac{7}{2}x+\left(\frac{7}{2}\right)^2=\frac{1}{2}+\frac{49}{4}$

$\left(x+\frac{7}{2}\right)^2=\frac{51}{4} \to \frac{57}{16}$

$\frac{7}{4}$

$x+\frac{7}{2}=\pm\sqrt{\frac{51}{4}}=\pm\frac{\sqrt{51}}{2} \to \frac{\sqrt{57}}{4}$

$\frac{7}{4}$

$$\boxed{x=-\frac{7}{2}\pm\frac{\sqrt{51}}{2}}$$

$\frac{7}{4} \qquad \frac{\sqrt{57}}{4}$

$x \in \left\{\frac{-7\pm\sqrt{51}}{2}\right\}$

GIGO

Bad

$\frac{1}{2}\cdot\frac{2}{2}+\frac{49}{4}=\frac{2+49}{4}=\frac{51}{4}$

$\sqrt{\frac{51}{4}}=\frac{\sqrt{51}}{\sqrt{4}}=\frac{\sqrt{51}}{2}$

$3\,\underline{)51}$
$17$

$\frac{1}{2}+\frac{49}{16}=\frac{1}{2}\cdot\frac{8}{8}+\frac{49}{16}$

$=\frac{8+49}{16}=\frac{57}{16}$

$\sqrt{\frac{57}{16}}=\frac{\sqrt{57}}{4}$

$x=-\frac{7}{4}\pm\frac{\sqrt{57}}{4}$

$$2x^2-7x-1=0$$

$$x^2-\frac{7}{2}x-\frac{1}{2}=0$$

$$\frac{\frac{7}{2}}{2}=\frac{7}{4} \rightsquigarrow \left(\frac{7}{4}\right)^2$$

$$\frac{1}{2}\cdot\frac{8}{8}+\frac{49}{16}=\frac{57}{16}$$

$$x^2-\frac{7}{2}x+\left(\frac{7}{4}\right)^2=\frac{1}{2}+\frac{49}{16}$$

$$\left(x-\frac{7}{4}\right)^2=\frac{57}{16}$$

$$x-\frac{7}{4}=\pm\frac{\sqrt{57}}{4}$$

$$x=\frac{7\pm\sqrt{57}}{4}$$

10

$x^2 + x - 56 = 0$

$a = 1, b = 1, c = -56$

$b^2 - 4ac = 1^2 - 4(1)(-56)$

$= 1 + 224$

$= 225$

$x = \frac{-1 \pm \sqrt{225}}{2(1)} = \frac{-1 \pm 15}{2}$

$x^2 + x + \frac{1}{4} = 56 + \frac{1}{4}$

$\frac{1}{2} \to \left(\frac{1}{2}\right)^2$

$\left(x + \frac{1}{2}\right)^2 = 56 + \frac{1}{4}$

$56 \times 4 = 224$

$\left(x + \frac{1}{2}\right)^2 = \frac{225}{4}$

$\frac{224}{4} + \frac{1}{4}$

$x + \frac{1}{2} = \pm\sqrt{\frac{225}{4}} = \pm\frac{15}{2}$

$x = -\frac{1 \pm 15}{2}$

$2.1x^2 + 6.5x - 4.6 = 0$

$21x^2 + 65x - 46 = 0$

$a = 21,\ b = 65,\ c = -46$

$b^2 - 4ac = 65^2 - 4(21)(-46) = 8089 \rightsquigarrow \sqrt{8089} \approx \boxed{89.93886813}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-65 \pm \sqrt{8089}}{2(21)} = \dfrac{-65 \pm \sqrt{8089}}{42}$

8

$a=4, b=-4, c=1$

$4x^2-4x+1$

$= (2x-1)^2$

$b^2-4ac=(-4)^2-4(4)(1) = 16-16 = 0$

$b^2-4ac = 0$ 1 real, repeated solution (Perfect square trinomial)

$b^2-4ac < 0$ 2 nonreal solutions

$b^2-4ac > 0$ 2 real solutions

( $b^2-4ac$ = Perfect square — Factors by ac method )

4, 9, 16, 25, ...

$-12x^2 + 7x + 13 = 0$

$b^2 - 4ac = 7^2 - 4(-12)(13)$

$= 49 + 624$

$= 673$

$$\begin{array}{r} {}^{2}\,48 \\ \underline{\times 13} \\ {}^{1}144 \\ \underline{480} \\ 624 \\ \underline{+\ 49} \\ 673 \end{array}$$

$12.25x^2 - 5.60x + .64 = 0$

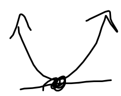
$1225x^2 - 560x + 64 = 0$

$a = 1225,\ b = -560,\ c = 64$

$b^2 - 4ac = (-560)^2 - 4(1225)(64) =$

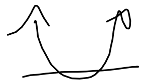
$x = \dfrac{560 \pm \sqrt{0}}{2(1225)} = \text{etc.}$

So, 1 solution is confirmed, here



(.2286, 0)

or



```
                  281600
(-560)²-4*1225*6
4
                       0
(-560)²-4*1225*6
4
                       0
█
```

18

$x^2+3x+16$

$a=1, b=3, 16$

$b^2-4ac = 3^2-4(1)(16)$

$= 9-64$

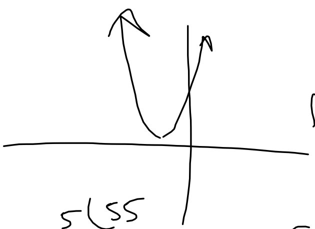
$= -55 < 0$

No real sol'n

Actual:

$x = \frac{-3 \pm \sqrt{-55}}{2(1)} = -\frac{3 \pm i\sqrt{55}}{2}$

$-\frac{3}{2} \pm \frac{\sqrt{55}}{2} i$



5(55

11

No x-intercept!

No real roots!

See? $b^2-4ac$

$= 3^2-4(1)(16)$

$= 9-64 = -55$

No real zeros!

19 Leading Coefficient is "1" Completing the square is efficient

$x^2 = \frac{8}{11}x + \frac{128}{121} = 0$

$(LCD)\left(x^2 - \frac{8}{11}x - \frac{128}{121}\right)$ $LCD = 11 \cdot 11$

$121x^2 - 88x - 128 = 0$ Quadratic Formula

$x^2 - \frac{8}{11}x + \left(\frac{4}{11}\right)^2 = \frac{128}{121} + \frac{16}{121}$

$\frac{\frac{8}{11}}{2} = \frac{8}{22} = \frac{4}{11} \rightarrow \left(\frac{4}{11}\right)^2$

$\left(x - \frac{4}{11}\right)^2 = \frac{144}{121}$ $\begin{array}{r} 128 \\ 16 \\ \hline 144 \end{array}$

$\left(x - \frac{4}{11}\right) = \pm\sqrt{\frac{144}{121}} = \pm \frac{\sqrt{144}}{\sqrt{121}} = \pm \frac{12}{11}$

$x - \frac{4}{11} = \pm \frac{12}{11}$

$x = \frac{4 \pm 12}{11}$ $\rightarrow \frac{16}{11}$, $\rightarrow \frac{-8}{11}$

$30x^2 + \sqrt{5}\,x - 1 = 0$

$b^2 - 4ac = (\sqrt{5})^2 - 4(30)(-1)$

$= 5 + 120$

$= 125$

$5 \,\underline{|\, 125}$
$5 \,\underline{|\, 25}$
$5$

$\sqrt{125} = \sqrt{5 \cdot 5 \cdot 5}$

$= 5\sqrt{5}$

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-\sqrt{5} \pm 5\sqrt{5}}{2(30)} =$

$\dfrac{-\sqrt{5} + 5\sqrt{5}}{60} = \dfrac{4\sqrt{5}}{60} = \dfrac{\sqrt{5}}{15}$

$\dfrac{-\sqrt{5} - 5\sqrt{5}}{60} = \dfrac{-6\sqrt{5}}{60} = -\dfrac{\sqrt{5}}{10}$

Check Domain: $\mathcal{D} = \{x \mid x \neq \pm 3\} = \mathbb{R} \setminus \{\pm 3\}$

21

$$\frac{x-12}{3-x} = \frac{x+33}{x+3} \qquad 3-x = -(-3+x) = -(x-3)$$

$$-\frac{x-12}{x-3} = \frac{x+33}{x+3} \qquad LCD = (x-3)(x+3)$$

$$-\left(\frac{x-12}{x-3}\right)\cdot\frac{x+3}{x+3} = \frac{x+33}{x+3}\cdot\frac{x-3}{x-3}$$

$$\frac{-(x^2 \overset{-9x}{+3x-12x} - 36)}{LCD} = \frac{x^2-3x+33x-99}{LCD}$$

$$\begin{array}{r} 99 \\ +36 \\ \hline 135 \end{array}$$

$$-x^2+9x+36 = x^2+30x-99$$

$$-2x^2-21x+135$$

$$a=-2,\ b=-21,\ c=135$$

$$b^2-4ac = (-21)^2-4(-2)(135)$$

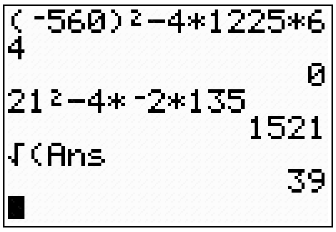
$$= 1521$$

$$\sqrt{1521} = 39$$

$$x = \frac{21 \pm 39}{2(-2)} \quad \nearrow \frac{21+39}{-4} = \frac{60}{-4} = -15 \qquad \searrow \frac{21-39}{-4} = \frac{-18}{-4} = \frac{9}{2}$$

3|135
3|45
3|15
5

$3\cdot 3\cdot 3\cdot 5\cdot(-2) = ac$



22

$s = -16t^2 + v_0 t + s_0$ $\frac{ft}{s}$

$v_0$ = initial velocity = $20\ \frac{ft}{s}$

metric version: $s = -4.9t^2 + v_0 t + s_0$ $\frac{m}{s}$

$s_0$ = initial height = 7 ft

$s = -16t^2 + 20t + 7 \overset{SET}{=} 0$

$a = -16,\ b = 20,\ c = 7$

$b^2 - 4ac = (20)^2 - 4(-16)(7)$

$= 848$

$\sqrt{848}$

$\approx 29.12043956$

```
                39
20²
               400
Ans+4*16*7
               848
√(Ans
       29.12043956
```

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-20 \pm \sqrt{848}}{2(-16)}$$

$\frac{-20 + \sqrt{848}}{-32}$

$\frac{-20 - \sqrt{848}}{-32} \approx 1.5$ when asked for nearest tenth.

```
Ans+4*16*7
               848
√(Ans
       29.12043956
(-20-√(848))/-32
       1.535013736
```

$\boxed{1.535013736 \approx t}$

23

$2^{-10}d^2s^3 - A^3 = 0$ Find $d$.

$2^{-10}d^2s^3 = A^3$

$\dfrac{2^{-10}d^2s^3}{2^{-10}s^3} = \dfrac{A^3}{2^{-10}s^3}$

$d^2 = \dfrac{A^3}{2^{-10}s^3} = \dfrac{2^{10}A^3}{s^3}$

$d = \pm\sqrt{\dfrac{2^{10}A^3}{s^3}} \rightarrow \pm\dfrac{2^5\sqrt{A^3}}{\sqrt{s^3}}$

$\sqrt{A^3} = A\sqrt{A}$

$A^3 = A \cdot A \cdot A$

$s^3 = s \cdot s \cdot s$

$= \pm\dfrac{2^5 A}{s}\sqrt{\dfrac{A}{s}} \cdot \dfrac{\sqrt{s}}{\sqrt{s}}$

$\dfrac{\sqrt{A}}{\sqrt{s}} \cdot \dfrac{\sqrt{s}}{\sqrt{s}} = \dfrac{\sqrt{As}}{\sqrt{s^2}} = \dfrac{\sqrt{As}}{s}$

$= \dfrac{2^5 A\sqrt{As}}{s^2}$ Prettying-up the symbolic answer.

$A = 816$
$s = 18.2$

$\dfrac{32\sqrt{816^3}}{\sqrt{18.2^3}} = 32\sqrt{\dfrac{816^3}{18.2^3}} \approx 9606.8$

```
32*√(816^3/18.2^
3)
      9606.786398
█
```

Parallel to $6x-y=3$ thru $(3,7)$ $y=m(x-x_1)+y_1$

$-y=-6x+3$

$y=6x-3$

$m=6$

$m_{\parallel}=6$

$y=6(x-3)+7$

$=6x-18+7$

$y=6x-11$

$-6x+y=-11$

$6x-y=+11$

1st → One returns 7% APR

2nd → " " 6% AR

Invested 8000 more in 2nd

9970 = Total Return (Interest)

Let $x$ = amt invested @ 7% (\$)

$y$ = " " " 6% (\$)

Total Interest is 9970

$.07x + .06y = 9970$

8000 more invested @ 6%

$y = x + 8000$

$.07x + .06(x + 8000) = 9970$

Solve for $x$.

Add 8000 to $x$ to ~~get~~ $y$.