MAT 121
70 Points

Writing Project #4 – Spring, 2019

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This Project is due by Test Day, whenever you take your test, either Wednesday, 4/24 or Thursday, 4/25. Neatness, Completeness and Margins count. Show all work.

Early Birds: Friday BEFORE the test. 10% bonus for doing so, with the added benefit of getting feedback BEFORE the test.

Face-to-Face Students: This Writing Project is due by the end of class, Wednesday, 4/24.

Online Students: If you can do a quality scan to a single, multi-page PDF, you may submit your work by e-mail *in the Course Shell*. Use the Classlist link in the Main Navbar, and attach it to a message to Harry Mills. I'm not accepting submissions to my steve.mills@aims.edu account.

1 Solve the system of linear equations $\begin{matrix} 3x-2y=12 \\ 5x+y=10 \end{matrix}$ in 3 ways:

$\frac{67+7}{70}$

a. (10 pts) Find the general vicinity of the solution by graphing the system. This should give you a general idea. Don't worry about it being super-accurate, although the more care you take, the better the estimate will be. Just graph the two lines by the intercept method. Supply the exact answer after you work parts b and c, below. I care much more about OPLs than tickmarks. OPLs are required. Tickmarks are not.

b. (10 pts) Use the Substitution Method

c. (10 pts) Use the Elimination Method.

2. (10 pts) Use Elimination to solve the independent system of linear equations: $\begin{matrix} x+2y+2z=7 \\ -2x+y+4z=0 \\ 2y+3z=6 \end{matrix}$.

3. Consider the dependent system of linear equations: $\begin{matrix} x+2y+2z=3 \\ 2x-y+7z=4 \\ -x+3y-5z=-1 \end{matrix}$.

a. (10 pts) Give the general solution. Be kind to your teacher and let z be free! That means, find an expression for x and y in terms of the variable z.

b. (10 pts) Give the particular solutions corresponding to $z=0$, $z=1$ and $z=-1$.

4. **The Underlying Assumption:** *All* of the techniques we learn for solving systems of linear equations are based on the *assumption* that the systems *have* solutions. So when we arrive at a false (*absurd!*) statement after a few elimination steps, the only explanation is that there was no solution in the first place*. Our incorrect assumption* led to something absurd, like $0=10$ or $0=-5$.

*... or you made a mechanical error and should check your work, just to make sure. Stay organized and always check your work.

Higher Learning: In higher mathematics, this is the most basic method of proving something is false: "Assume it's true and conclude something absurd (like '0 = 1')." It's important that you realize what's happening when you arrive at those absurdities at the end of a perfectly logical and legal sequence of moves. That said, let me *finally* get to the question:

(10 pts) **Your Task:** Show that the dependent system of linear equations

$$\begin{aligned} x + 2y + 2z &= 3 \\ 2x - y + 7z &= 4 \\ -x + 3y - 5z &= 0 \end{aligned}$$

has no solution. I expect to see the word "absurd" in your discussion.

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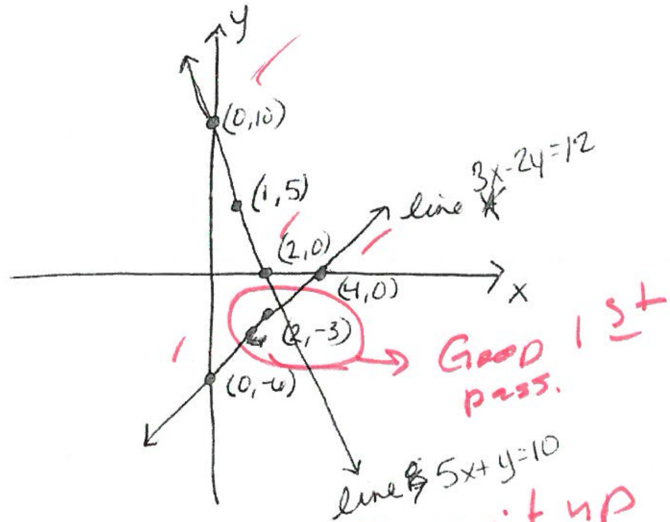
1. A) $3x-2y=12$

$y=mx+b$

$\frac{-2y}{-2}=\frac{-3x}{-2}+\frac{12}{-2}$

$y=\frac{3}{2}x-6$

↑ slope ↑ y-intercept



Firm it up after 1b & 1c.

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~~B)~~ $5x+y=10$

$-5x \quad -5x$

$y=-5x+10$

↑ slope ↑ y-intercept

~~B) $3x-2y=12$~~

~~$5x+y=10$~~ $\rightarrow y=-5x+10$

~~$3(-5x+10)-$~~

Re-do ↓

B) $3x-2y=12$

$5x+y=10 \rightarrow y=-5x+10$

$3x-2(-5x+10)=12$

$3x+10x-20=12$

$+20 \quad +20$

$\frac{13x}{13}=\frac{32}{13}$ $\quad x=\frac{32}{13}$

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1) B)(cont)

$y = -5x+10$

$y = -5\left(\frac{32}{13}\right)+10$

$y = -\frac{30}{13}$

$\boxed{\left\{\left(\frac{32}{13}, \frac{-30}{13}\right)\right\}}$

10

c) 1) $3x-2y=12$

2) $5x+y=10$

2E2) $10x+\cancel{2y}=20$

E1) $+\ 3x-\cancel{2y}=12$

$\frac{13x}{13} = \frac{32}{13}$

$\boxed{x = \frac{32}{13}}$

E2) $5\left(\frac{32}{13}\right)+y=10$

$\frac{160}{13}+y=10$

$-\frac{160}{13} \qquad -\frac{160}{13}$

$\boxed{y = -\frac{30}{13}}$

$\boxed{\left\{\left(\frac{32}{13}, -\frac{30}{13}\right)\right\}}$

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2) E1) $x + 2y + 2z = 7$

E2) $-2x + y + 4z = 0$

E3) $2y + 3z = 6$

$$\left.\begin{array}{l} \text{2E1)}\ 2x + 4y + 4z = 14 \\ \text{E2)}\ -2x + y + 4z = 0 \\ \text{E3)}\ \quad 2y + 3z = 6 \\ \hline \quad 7y + 11z = 20 \\ \\ 7y + 11z = 20 \end{array}\right\}$$

Not a very efficient method, generally. Work with 2 Equations at a time

&

Gaussian Elimination should ~~by~~ be very systematiz & ~~do~~ involve more eliminating

&

You're doing a mostly "substitution method. See my solutions

$7y + 11z = 20$

$-11z \quad -11z$

$\frac{7y}{7} = \frac{-11z}{7} + \frac{20}{7}$

$y = -\frac{11}{7}z + \frac{20}{7}$

E3) $2\left(-\frac{11}{7}z + \frac{20}{7}\right) + 3z = 6$

$-\frac{22}{7}z + \frac{40}{7} + 3z = 6$

$-\frac{1}{7}z + \frac{40}{7} = 6$

$\frac{-\frac{1}{7}z}{-\frac{1}{7}} = \frac{\frac{2}{7}}{-\frac{1}{7}}$

$\boxed{z = -2}$

E3) $2y + 3(-2) = 6$

$2y - 6 = 6$

$\quad +6 \quad +6$

$2y = 12$

$\boxed{y = 6}$

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2)(cont)

~~EX~~

E1) $x+2y+2z=7$

$x+2(6)+2(-2)=7$

$x+12-4=7$

$x+8=7$
$\quad -8 \quad -8$

$\boxed{x=-1}$

$\boxed{\{(-1,6,-2)\}}$

3A)

9.5

3.) E1) $x+2y+2z=3$
E2) $2x-y+7z=4$
E3) $-x+3y-5z=-1$

E1) $x+2y+2z=3$
E3) $-x+3y-5z=-1$

$5y-3z=2$
$\quad +3z \quad +3z$

$\frac{5y}{5}=\frac{3z}{5}+\frac{2}{5}$

$\boxed{y=\frac{3}{5}z+\frac{2}{5}}$

Do $-2E1+E2$:

$-2x-4y-4z=-6$
$2x-y+7z=4$

$-5y+3z=-2$

Th.3 with that

Give new system:

$x+2y+2z=3$
$-5y+3z=-2$
$5y-3z=2$

E1) $x+2y+2z=3$
2E2) $4x-2y+14z=8$

$5x+16z=11$
$\quad -16z \quad -16z$

$\frac{5x}{5}=\frac{-16z}{5}+\frac{11}{5} \rightarrow \boxed{x=-\frac{16}{5}z+\frac{11}{5}}$

Your methods are pretty haphazard. You should want to be Systematic for bigger stuff in the future.

See Solutions.

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3(cont)A) $\boxed{\left\{\left(-\frac{16}{5}z+\frac{11}{5},\ \frac{3}{5}z+\frac{2}{5},\ z\right)\right\}}$ z is any real number

3.B) $z=0$ $\boxed{\left\{\left(\frac{11}{5},\frac{2}{5},0\right)\right\}}$

$z=1$ $\left(-\frac{16}{5}\cdot 1+\frac{11}{5},\ \frac{3}{5}\cdot 1+\frac{2}{5},\ 1\right)$

$\boxed{\{(-1,1,1)\}}$

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$z=-1$ $\left(-\frac{16}{5}\cdot -1+\frac{11}{5},\ \frac{3}{5}\cdot -1+\frac{2}{5},\ -1\right)$

$\boxed{\left\{\left(\frac{27}{5},-\frac{1}{5},-1\right)\right\}}$

4.) E1) $x+2y+2z=3$
E2) $2x-y+7z=4$
E3) $-x+3y-5z=0$

E1) $\cancel{x}+2y+2z=3$
E3) $\cancel{-x}+3y-5z=0$
$5y-3z=3$

-2E1) $\cancel{-2x}-4y-4z=-6$
E2) $\cancel{2x}-y+7z=4$
$-5y+3z=-2$

Write the new system

$x+2y+2z=3$
$5y-3z=3$
$-5y+3z=-2$

Then do one more step"

E2 $5y-3z=3$
+ E3 $-5y+3z=-2$
$0=1$?!

Be systematic

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4.) (cont)

E1 + E3) $5y - 3z = 3$

−2E1 + E2) $-\cancel{5}y + \cancel{3}z = -2$

$0 + 0 = 1$

$\boxed{0 = 1 \text{ absurd! no solution}}$

These equations have no solution

~~OD~~ OLD Instructions

These are now E2 & E3 in a new (equivalent) system

that I wrote down on previous page.

Systematizi: Top to bottom Left to right.

OK

9.5

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