

Test 1, Fall, 2014 - Run-Through and Other Notes

Find all real or imaginary solutions in #s 1 - ~~4~~ 4.

1. (5 pts) $-3x - 7 = 2x + 21$

$$\begin{array}{r} -2x + 7 = -2x + 7 \\ \hline -5x = 28 \end{array}$$

$$x = -\frac{28}{5}$$

2. (5 pts) $\frac{3}{5}x - \frac{2}{3} = \frac{5}{6}$ LCD = 5 · 3 · 2

$$\frac{3x}{5} \cdot \frac{6}{6} - \frac{2}{3} \cdot \frac{10}{10} = \frac{5}{6} \cdot \frac{5}{5}$$

$$\frac{18x - 20}{\text{LCD}} = \frac{25}{\text{LCD}}$$

$$18x - 20 = 25$$

$$18x = 45$$

$$x = \frac{45}{18} = \frac{15}{6} = \frac{5}{2}$$

$$x \in \left\{ \frac{5}{2} \right\}$$

3. (5 pts) $9x^2 = 7$ No x' -term
Square-root property

$$9x^2 = 7$$

$$x^2 = \frac{7}{9}$$

$$\sqrt{x^2} = \sqrt{\frac{7}{9}}$$

$$|x| = \sqrt{\frac{7}{9}}$$

$$x = \pm \sqrt{\frac{7}{9}} = \pm \frac{\sqrt{7}}{\sqrt{9}} = \pm \frac{\sqrt{7}}{3}$$

$$x \in \left\{ \pm \frac{\sqrt{7}}{3} \right\}$$

4. (5 pts) $5x^2 - 10x + 17 = 0$ (Leave your final answer in simplified radical form.)

$$a=5, b=-10, c=17$$

$$\Rightarrow b^2 - 4ac = (-10)^2 - 4(5)(17)$$

$$= 100 - 340$$

$$= -240$$

$$\sqrt{-240} = ?$$

$$\sqrt{240} = 2 \cdot 2 \sqrt{15}$$

$$= 4\sqrt{15}$$

$$\sqrt{-240} = 4i\sqrt{15}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{10 \pm 4i\sqrt{15}}{2(5)} = \frac{10 \pm 4i\sqrt{15}}{10} = \frac{2(5 \pm 2i\sqrt{15})}{10} = \frac{5 \pm 2i\sqrt{15}}{5}$$

$$\frac{5}{5} \pm \frac{2i\sqrt{15}}{5} = 1 \pm \frac{2\sqrt{15}}{5}i = a + bi$$

Looks legit.

$$\frac{5 \pm 2i\sqrt{15}}{5}$$

5. (10 pts) Compute the discriminant for each of the following equations and tell me what it tells you about the solutions of the equations, *without having to solve them*, i.e., don't solve.

a. $10x^2 + 81x - 133 = 0$

$$b^2 - 4ac \qquad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 10, b = 81, c = -133 \Rightarrow$

$$b^2 - 4ac = (81)^2 - 4(10)(-133)$$

$$= 11881 = 109^2 \text{ is}$$

a perfect square, so

2 solutions AND

it factors in conventional way
AND its solutions are
rational

→ Real!

$(10)(-133) = -1330 \rightarrow$ Magic #

81 Subtract to hit the 81!

- 81 = 82 - 1 -82 Higher!
- = 83 - 2 -166 Higher!
- = 91 - 10 -910 higher!
- = 101 - 20 -2020 Lower!
- = 96 - 15 -1440 Lower!
- = 94 - 13 -1222 Higher!
- = 95 - 14 -1330! Sweet!

So,

$$10x^2 + 81x - 133$$

$$= 10x^2 + 95x - 14x - 133$$

$$= 5x(2x + 19) - 7(2x + 19)$$

$$= (2x + 19)(5x - 7)$$

$x \in \left\{ -\frac{19}{2}, \frac{7}{5} \right\}$

b. $25x^2 - 50x + 28 = 0$

$a = 25, b = -50, c = 28$

$$b^2 - 4ac = (-50)^2 - 4(25)(28)$$

$$= \text{negative}$$

2 nonreal solutions

6. (10 pts) Solve $x^2 + 12x - 17 = 0$ by completing the square.

$$\begin{aligned}x^2 + 12x + 6^2 &= 17 + 36 \\ \downarrow \\ \frac{12}{2} = 6 &\rightarrow 6^2 \\ (x+6)^2 &= 53 \\ \sqrt{(x+6)^2} &= \sqrt{53} \\ |x+6| &= \sqrt{53} \\ x+6 &= \pm\sqrt{53} \\ x &= -6 \pm \sqrt{53} \\ x &\in \{-6 \pm \sqrt{53}\}.\end{aligned}$$

7. (5 pts) Find an equation of the line through (2,5) and (3,-7). Point-slope is preferred, but not required.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 5}{3 - 2} = \frac{-12}{1} = -12$$

(x_1, y_1) (x_2, y_2)

$$y = m(x - x_1) + y_1$$

$$y = -12(x - 2) + 5$$

$$y = -12(x - 3) - 7$$

8. (5 pts) Find an equation of the line thru (8,-6) that is parallel to the line $y = 3x - 11$.

$$y = m(x - x_1) + y_1$$

$$y = 3(x - 8) - 6$$

$$\rightarrow m_{||} = m$$

$$\rightarrow m = 3 = m_{||}$$

9. (5 pts) Find an equation of the line thru (8,-6) that is perpendicular to the line $y = 3x - 11$.

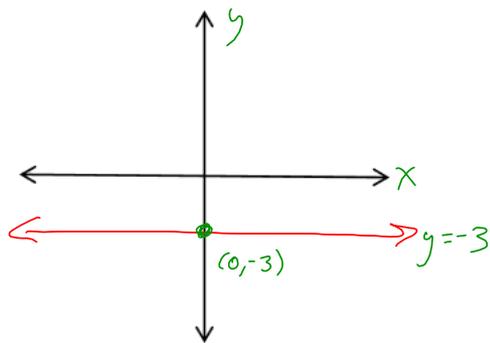
$$y = m(x - x_1) + y_1$$

$$y = -\frac{1}{3}(x - 8) - 6$$

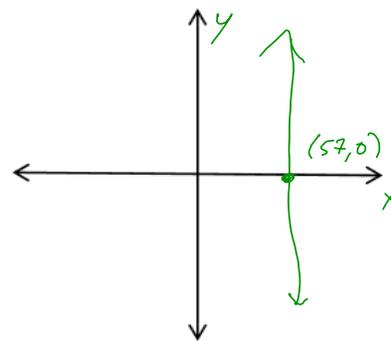
$$\rightarrow m_{\perp} = -\frac{1}{m}$$

$$\rightarrow m = 3 \Rightarrow m_{\perp} = -\frac{1}{3}$$

10. (5 pts) Sketch the graph of the line $y = -3$



11. (5 pts) Sketch the graph of the line $x = 57$



Solve the inequalities. Give your answer as a set and as an interval. You may want to use a number line graph to help you write your answer, but it is not required.

12. (5 pts) $-5x - 4 > 34$

$$-5x > 38$$

$$\left\{ x \mid x < \frac{38}{-5} \right\}$$

$$= \left(-\infty, -\frac{38}{5} \right)$$

$$= \left(-\infty, -\frac{38}{5} \right)$$

$|A| \geq B \Rightarrow$
 $A \geq B$ OR $A \leq -B$

13. (5 pts) $|2x - 7| \geq 8$

$2x - 7 \geq 8$ OR $2x - 7 \leq -8$

$2x \geq 15$ OR $2x \leq -1$

$\{x \mid x \geq \frac{15}{2} \text{ OR } x \leq -\frac{1}{2}\}$



$= (-\infty, -\frac{1}{2}] \cup [\frac{15}{2}, \infty)$

Set-builder

$|A| < B \Rightarrow$

$A < B$ AND $A > -B$

14. (5 pts) $|2x - 7| < 8$

$2x - 7 < 8$ AND $2x - 7 > -8$

$2x < 15$ AND $2x > -1$

$\{x \mid x < \frac{15}{2} \text{ AND } x > -\frac{1}{2}\}$



$= (-\frac{1}{2}, \frac{15}{2})$

15. (5 pts) $|2x - 3| > -7$

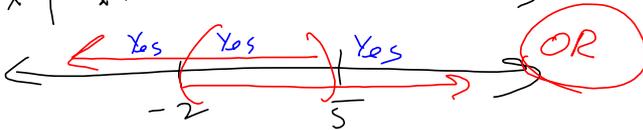
\mathbb{R} STOP!

If not, then here's how it'd go

$2x - 3 > -7$ OR $2x - 3 < 7$

$2x > -4$ OR $2x < 10$

$\{x \mid x > -2 \text{ OR } x < 5\}$



$= (-\infty, \infty) = \mathbb{R}$

16. (5 pts) $|2x - 3| \leq -7$

\emptyset

17. (5 pts) How much 20% nitrate solution must be added to 50 liters of 50% nitrate solution to obtain a 34% nitrate solution?

	Vol	Pure NO_3
20% NO_3	x	$.2x$
50% NO_3	50	$(.5)(50)$
Mixture	$x+50$	$.2x + .5(50) = .34(x+50)$

Let x = amt of 20% solution (in L).

How much 20% Nitric Acid solution must be added to a 50% Nitric Acid solution to obtain 100 liters of 34% solution?

Let x = amt of 20% NO_3 sol'n (in L)
 y = " " 50% NO_3 " (in L)

TOTAL VOL. $x + y = 100$
 Pure acid $.2x + .5y = .34(100)$

18. (5 pts) John can do a job in 12 hours that takes Bob 16 hours. Suppose John sleeps in on the day they were to work together and shows up 2 hours late. How many hours does Bob end up working, if they finish the job together? How many hours does John end up working that day?

$$\begin{array}{l} \text{Amt of work} \\ \text{By Bob} \end{array} + \begin{array}{l} \text{Amt of work} \\ \text{by John} \end{array} = 1 \text{ Job done.}$$

$$\left(\frac{1 \text{ Job}}{16 \text{ hr}}\right)(x \text{ hrs}) + \left(\frac{1 \text{ Job}}{12 \text{ hr}}\right)(y \text{ hours}) = 1$$

Let
 $x = \# \text{ of hours Bob works}$
 $y = \text{" " " John works}$

$$y = x - 2$$

$$\frac{1}{16}x + \frac{1}{12}(x-2) = 1$$

BONUS (5 pts) Re-write the function $f(x) = x^2 - 8x - 5$ in the form $f(x) = a(x - h)^2 + k$.
State the vertex of this parabola.

BONUS (5 pts) Re-write the function $g(x) = 5x^2 + 10x - 19$ in the form
 $g(x) = a(x - h)^2 + k$. State the vertex of this parabola.

