2. Let $f(x)=\frac{x-6}{x-12}$ and $g(x)=\sqrt{x+4}$ and .

TEST 2, Fall, 2015 #2

a. (5 pts) Write the function $\frac{f}{g}$. Do not simplify.

b. (5 pts) What is the domain of $\frac{f}{g}$? Give your answer in set notation and interval notation.

Need $\mathcal{D}(f)$ & $\mathcal{D}(g)$ to do $\mathcal{D}(\frac{f}{g})$

Look for separate questions: $\mathcal{D}(f)$, $\mathcal{D}(g)$

c. (5 pts) Write the function $f \circ g$. Do not simplify.

d. (5 pts) What is the domain of $f \circ g$? Give your answer in set notation and interval notation.

Domain

Division by zero $\frac{A}{B}$ Need $B \neq 0$.

Square root of negative. $\sqrt{B}$ Need $B \geq 0$.

$g(x)=\sqrt{x+4}$ Need: $x+4 \geq 0$ Set-builder.

$\mathcal{D}(g) = \{x \mid x \geq -4\}$

Number line graph is the assist.

$= [-4, \infty)$ Interval.

$f(x)=\frac{x-6}{x-12}$ Need: $x-12 \neq 0$

$\mathcal{D}(f) = \{x \mid x \neq 12\}$

Basic Domain.

$= (-\infty, 12) \cup (12, \infty)$

$x \in$ here OR $x \in$ here

$\frac{f}{g} = \frac{f(x)}{g(x)} = \frac{\frac{x-6}{x-12}}{\sqrt{x+4}}$ STOP!

$\mathcal{D}(\frac{f}{g}) = \{x \mid x$ is something $\frac{f}{g}$ can eat.$\}$

$= \{x \mid f$ is happy and $g$ is happy AND $g(x) \neq 0\}$

$= \{x \mid x \neq 12$ and $x \geq -4$ and $x \neq -4\}$

$x > -4$

$g(x) \neq 0$

$\sqrt{x+4} \neq 0$

$\sqrt{x+4}^2 \neq 0^2$

$x+4 \neq 0$

$x \neq -4$

NO NO YES NO YES AND

$-4$ $12$

$= (-4, 12) \cup (12, \infty)$

Domain of $f \circ g$ next time, using this example.

$\mathcal{D}(g) = \{x \mid x \geq -4\}$ $\mathcal{D}(f) = \{x \mid x \neq 12\}$

$f(x) = \frac{x-6}{x-12}$ $g(x) = \sqrt{x+4}$ Feeding $g(x)$ to $f(x)$:

(c) Write $f \circ g$ : $(f \circ g)(x) = f(g(x)) = f(\sqrt{x+4}) = \frac{\sqrt{x+4}-6}{\sqrt{x+4}-12}$

(d) Find $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \geq -4 \text{ and } g(x) \neq 12\}$

$= \{x \mid x \geq -4 \text{ and } \sqrt{x+4} \neq 12\}$

$= \boxed{\{x \mid x \geq -4 \text{ and } x \neq 140\}}$ Set-builder

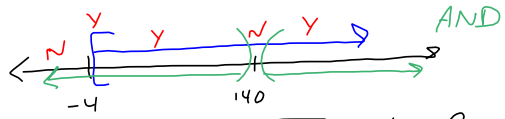
$\sqrt{x+4} \neq 12$

$\sqrt{x+4}^2 \neq 12^2$

$x+4 \neq 144$

$x \neq 140$

Number-line graph is the bridge.

$=$ 


$= \boxed{[-4, 140) \cup (140, \infty)}$ Interval

We just got done feeding $g$ to $f$, with $f \circ g = f(g(x))$. Now we turn it around and try feeding $f$ into $g$, with the composition

$g \circ f = g(f(x))$

$\mathcal{D}(g) = \{x \mid x \geq -4\}$ $\mathcal{D}(f) = \{x \mid x \neq 12\}$

$f(x) = \frac{x-6}{x-12}$ $g(x) = \sqrt{x+4}$

$g \circ f = g(f(x)) = g\left(\frac{x-6}{x-12}\right) = \sqrt{\frac{x-6}{x-12} + 4}$

$\mathcal{D}(g \circ f) = \{x \mid x \in \mathcal{D}(f) \text{ and } f(x) \in \mathcal{D}(g)\}$

$= \{x \mid x \neq 12 \text{ and } \frac{x-6}{x-12} \geq -4\}$

$= \{x \mid x \neq 12 \text{ and } x \in (12, 14]\}$

$= (12, 14] = \mathcal{D}(g \circ f)$

$x \in (12, 14]$ means $-12 < x \leq 14$

Scratch

$\frac{x-6}{x-12} \geq 4$

$\frac{x-6}{x-12} - 4 \geq 0$

$\frac{x-6}{x-12} - \frac{4}{1}\left(\frac{x-12}{x-12}\right) \geq 0$

$\frac{x-6-4(x-12)}{x-12} \geq 0$

$\frac{x-6-4x+48}{x-12} \geq 0$

$\frac{-3x+42}{x-12} \geq 0$ $x \neq 11$

$-3x+42 = 0$ $x-12 = 0$

$-3x = -42$ $x = 12$

$x = \frac{-42}{-3} = 14 = x$

Another way to analyze

$\frac{-3x+42}{x-12} \geq 0$

Breaks it into intervals: $(-\infty, 12)$ $(12, 14)$ $(14, \infty)$

$\frac{-3(11)+42}{11-12} = \frac{9}{-1} = -9$

$x = 13$ $\frac{-3(13)+42}{13-12} = \frac{3}{1} = +9$

$x = 15$ $\frac{-3(15)+42}{15-12} = \frac{-3}{3} = -1$

Coming Soon:

H.A.: $\frac{-3x+42}{x-12}$ $x \to \text{Big}$ $\frac{-3x}{x} = -3$

TEST VALUE METHOD

NEED $\geq 0$

$= (12, 14] =$