1. Consider the relation $f = \{(-2,3),(1,5),(2,3),(3,-2)\}$.

   a. (5 pts) Is f a function? Yes. No repetitions in 1^{st} coordinate

   b. (5 pts) What is the domain of f ? $\mathcal{D}(f) = \{-2, 1, 2, 3\}$

   c. (5 pts) What is the range of f ? $\mathcal{R}(f) = \{3, 5, 3, -2\} = \{3, 5, -2\}$

   d. (5 pts) Is f one-to-one? If not, explain why not.

      No. $(-2,3)$ & $(2,3)$ share the same 2^{nd} coordinate

Further Example: $f = \{(-2,3),(1,5),(2,\cancel{3}\,4),(3,-2)\}$

This IS 1-to-1 func. Silly! Messed up in my prob. construction.

$\{(-2,3),(1,5),(2,4),(3,-2)\}$

3. Let $f(x)=\sqrt{x-2}$ and $g(x)=\dfrac{x-7}{x+5}$.

a. (5 pts) Write the function $\dfrac{f}{g}$. Do not simplify.

$$\boxed{\dfrac{\sqrt{x-2}}{\dfrac{x-7}{x+5}}}$$ stop!

simplified:

$$\frac{(x+5)\sqrt{x-2}}{x-7}$$

b. (5 pts) What is the domain of $\dfrac{f}{g}$?

$\mathcal{D}(f)=[2,\infty)$

$\mathcal{D}(g)=\{x \mid x\neq -5\}$

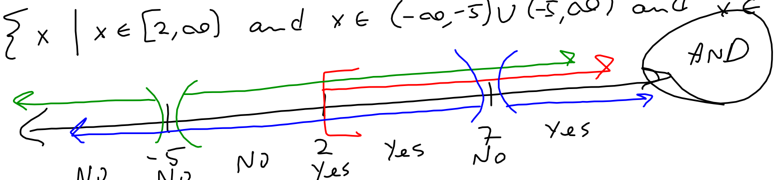
$=(-\infty,-5)\cup(-5,\infty)$

$\mathcal{D}=\{x \mid x\in\mathcal{D}(f) \text{ and } x\in\mathcal{D}(g) \text{ and } g(x)\neq 0\}$

$=\boxed{\{x \mid x\geq 2 \text{ and } x\neq -5 \text{ and } x\neq 7\}}$

OPTIONAL WAY

$=\{x \mid x\in[2,\infty) \text{ and } x\in(-\infty,-5)\cup(-5,\infty) \text{ and } x\in(-\infty,7)\cup(7,\infty)\}$

$=$ 

$=\boxed{[2,7)\cup(7,\infty)}$

$g(x)\neq 0$ SCRATCH

$\dfrac{x-7}{x+5}\neq 0$

$x-7\neq 0$

$x\neq 7$

3. Let $f(x)=\sqrt{x-2}$ and $g(x)=\dfrac{x-7}{x+5}$.

$\mathcal{D}(f) = [2,\infty) = \{x \mid x \ge 2\}$

c. (5 pts) Write the function $f \circ g$. Do not simplify.

$(f\circ g)(x) = f(g(x))$

$$= \boxed{\sqrt{\frac{x-7}{x+5} - 2}}$$

STOP!

simplified

$$\sqrt{\frac{x-7-2(x+5)}{x+5}}$$

$$= \sqrt{\frac{x-7-2x-5}{x+5}}$$

$$= \sqrt{\frac{-x-12}{x+5}}$$

$$= \sqrt{-\frac{x+12}{x+5}}$$

d. (5 pts) What is the domain of $f \circ g$?

Nice Bonus for Test 2!

$\mathcal{D} = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \ne -5 \text{ and } \frac{x-7}{x+5} \ge 2\}$

Domain of composition NOT bonus, but this analysis is bonus

Scratch for $\frac{x-7}{x+5} \ge 2$:

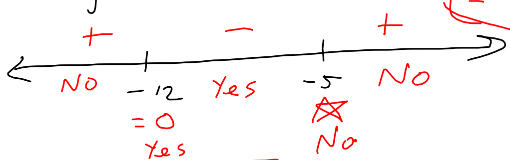
$\frac{x-7}{x+5} - 2 \ge 0$

$-\frac{(x+12)}{x+5} \ge 0$

$\frac{x+12}{x+5} \le 0$

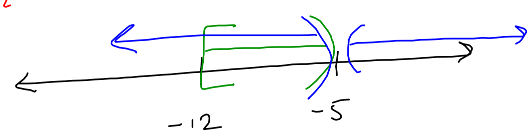
Key #s: $x=-12$, $x=-5$

≤ 0



$= \boxed{[-12,-5)}$

$= \{x \mid x \ne -5 \text{ and } -12 \le x < -5\}$

$=$ 

$= [-12,-5)$

Domain of Composition with more suitable example, at least for Test 2

$f(x) = \dfrac{1}{x-3}$, $g(x) = \sqrt{x+5}$

$\mathcal{D}(f) = \mathbb{R} \setminus \{3\}$

$= (-\infty, 3) \cup (3, \infty)$

$(f \circ g)(x) = \dfrac{1}{\sqrt{x+5} - 3}$ Stop here on test.

$\mathcal{D}(g) =$ Need $x+5 \geq 0$

$= \{x \mid x \geq -5\}$

$= [-5, \infty)$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

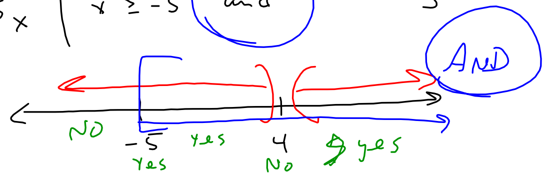
$= \{x \mid x \geq -5 \text{ and } \sqrt{x+5} \neq 3\}$

$\sqrt{x+5} \neq 3$

$x+5 \neq 9$

$x \neq 4$

$= \{x \mid x \geq -5 \text{ and } x \neq 4\}$

$=$ 

$= [-5, 4) \cup (4, \infty)$

4. (5 pts) Simplify the difference quotient for $f(x) = 2x^2$ $f(x+h) \neq f(x) + h$

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 2x^2}{h} = \frac{2(x^2+2xh+h^2) - 2x^2}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h} = \frac{4xh + 2h^2}{h} = \frac{h(4x+2h)}{h} = \boxed{4x+2h}$$

Bonus:

$$4x+2h \xrightarrow{h \to 0} 4x$$

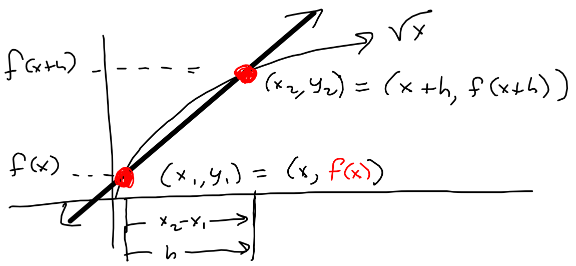
Bonus (5 pts) Pass to the limit as *h* approaches zero, and show me some calculus to go with #4.

Bonus:

$$4x+2h \xrightarrow{h \to 0} 4x$$

5. (5 pts) Draw a picture for the difference quotient for $f(x) = \sqrt{x}$. Describe what the difference quotient represents, in words. Do not simplify your difference quotient. That's a bonus problem, later on.

$\frac{f(x+h) - f(x)}{h}$ = slope of line between two points on a curve.



$$\frac{f(x+h) - f(x)}{h} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

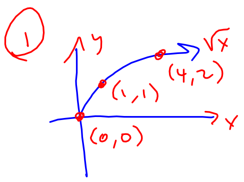
6. Let $g(x) = -\sqrt{10-5x}+7$. $= -\sqrt{-5(x-2)} + 7$

   a. (10 pts) Sketch the graph of $g(x)$, by transforming the basic function $f(x) = \sqrt{x}$. I want to see 3 points labeled in the graph of g – preferably starting with $(0,0)$, $(1,1)$ and $(4,2)$ – and track where those points are moved to after every step, as demonstrated in class.

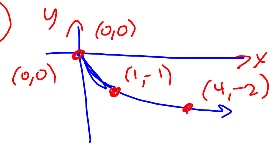
$10-5x = -5(-2+x) = -5(x-2)$

$$\text{①}\ \sqrt{x} \longrightarrow \text{②}\ -\sqrt{x} \longrightarrow \text{③}\ -\sqrt{-5x} \longrightarrow \text{④}\ -\sqrt{-5(x-2)} \longrightarrow \text{⑤}\ -\sqrt{-5(x-2)}+7$$

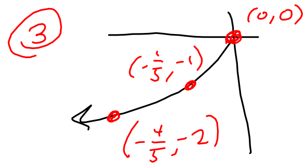
$(x,-y)$ $\quad (-\frac{1}{5}x, y)$ $\quad (x+2, y)$ $\quad (x, y+7)$

① 

$f(x) = \sqrt{x}$

② 

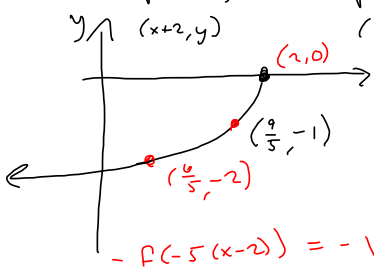
$-f(x) = -\sqrt{x}$

③ 

$-f(-5x) = -\sqrt{-5x}$

$$\text{④}\ -\sqrt{-5(x-2)} \longrightarrow \text{⑤}\ -\sqrt{-5(x-2)}+7$$

$(x+2,y)$ $\quad (x,y+7)$

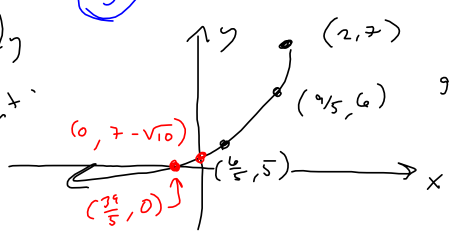


$-f(-5(x-2)) = -\sqrt{-5(x-2)}$

$-\frac{1}{5}+2 = \frac{-1+10}{5} = \frac{9}{5}$

$-\frac{4}{5}+2 = \frac{-4+10}{5} = \frac{6}{5}$

Probably want x-int.

⑤ 

$g(x) = -f(-5(x-2))+7$

$= -\sqrt{-5(x-2)}+7$

y-axis position

see y-int:

$g(0) = -\sqrt{-5(0-2)}+7$

$= -\sqrt{10}+7$

> 0

x-int: $g(x) = 0$

$-\sqrt{-5(x-2)}+7 = 0$

$-\sqrt{-5x+10} = -7$

$\sqrt{-5x+10} = 7$

$-5x+10 = 49$

$-5x = 39$

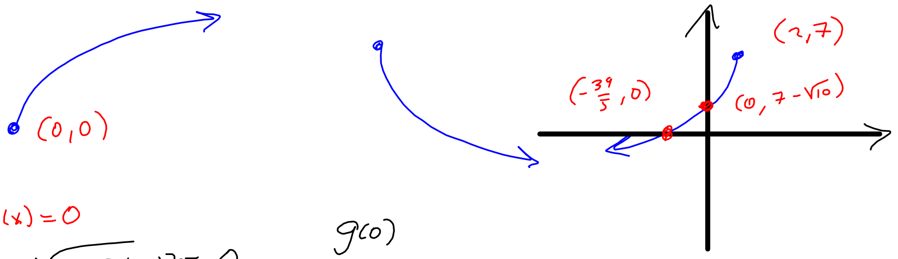
$x = -\frac{39}{5}$ Not agreeing with picture!

$g(x) = -\sqrt{10-5x} + 7$ Graphing Quick 'n' dirty.

$= -\sqrt{-5(x-2)} + 7$

- Vertical Flip
- Horizontal Flip
- Right 2
- Up 7

Use intercepts to locate axes' position.



$g(x)=0$

$-\sqrt{10-5x} + 7 = 0$

$-\sqrt{10-5x} = -7$

$\sqrt{10-5x} = 7$

$10-5x = 49$

$-5x = 39$

$x = -\frac{39}{5}$

$(-\frac{39}{5}, 0)$

Neg.

$g(0)$

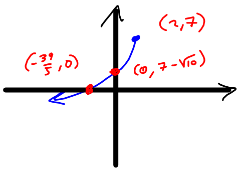
$-\sqrt{10-5(0)} + 7$

$= -\sqrt{10} + 7$

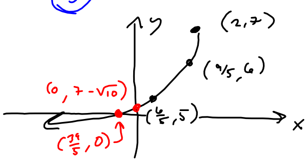
$(0, 7-\sqrt{10})$

pos.

b. (5 pts) State the domain and range of $g(x)$, based on your final graph.



5



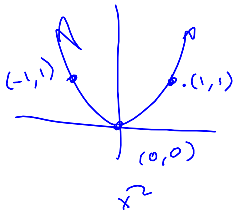
$(-\infty, 2] = D$

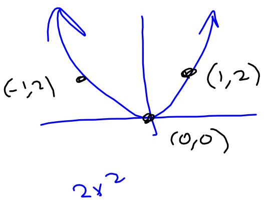
$(-\infty, 7] = R$

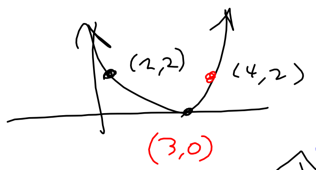
c. (5 pts) Find the x- and y-intercept of $g(x)$, and label them, clearly, on the graph.

7. (10 pts) Sketch the graph of $r(x)=2(x-3)^2-5$ by transforming the basic function $f(x)=x^2$. I want to see 3 points labeled in the graph of f, and I want you to track where those points are moved to after every step, as demonstrated in class.

$x^2 \longrightarrow 2x^2 \longrightarrow 2(x-3)^2 \longrightarrow 2(x-3)^2-5$





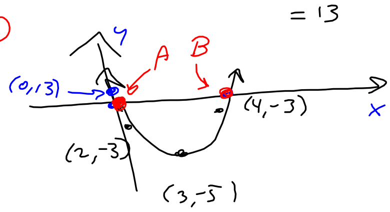


$g(0) = 2(-3)^2-5$

$= 18-5$

$= 13$

Too Cramped!



8. (5 pts) Find the x- and y-intercepts and add them to your final sketch, above. For x-intercept, leave final answer in simplified radical form.

$2(x-3)^2-5=0$

$2(x-3)^2=5$

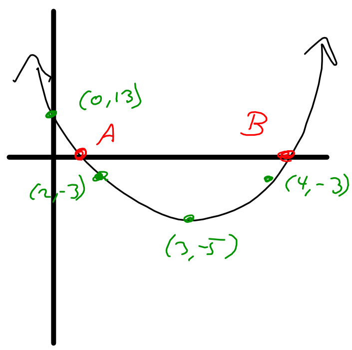
$(x-3)^2=\frac{5}{2}$

$x-3=\pm\sqrt{\frac{5}{2}}$

$x=3\pm\sqrt{\frac{5}{2}} = 3\pm\frac{\sqrt{10}}{2}$

$A=\left(3-\sqrt{\frac{5}{2}}, 0\right)$

$B=\left(3+\sqrt{\frac{5}{2}}, 0\right)$



$\sqrt{2.5}$

$3-\sqrt{\frac{5}{2}} > 0$

9. (5 pts) Prove that $\dfrac{x+1}{x-3}$ is one-to-one.

$x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$

or Contrapositive

$f(x_1) = f(x_2) \implies x_1 = x_2$

Suppose $f(x_1) = f(x_2)$

$$\frac{x_1+1}{x_1-3} = \frac{x_2+1}{x_2-3}$$

$$(x_2-3)(x_1+1) = (x_2+1)(x_1-3)$$

$$x_2x_1 + x_2 - 3x_1 - 3 = x_2x_1 - 3x_2 + x_1 - 3$$

$$x_2 - 3x_1 = x_1 - 3x_2$$

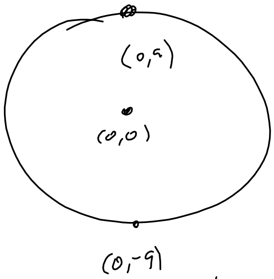
$$4x_2 = 4x_1$$

$$x_2 = x_1 \quad \blacksquare$$

10. (5 pts) Suppose y is jointly proportional to the square of x and the cube of z, and inversely proportional to u and the square root of w. Write an equation for this relationship between y, x, z, u, and w.

$$\boxed{y = k\,\frac{x^2 z^3}{u\sqrt{w}}}$$

11. (5 pts) Explain why $x^2 + y^2 = 81$ does *not* define y as a function of x.



It's a circle!
Flunks vertical line test.

Algebraic Way:
Solve eq'n for y.

$$y^2 = 81 - x^2$$

$$y = \pm\sqrt{81 - x^2}$$

Look! 2 y-values for $x = 0$, e.g.!

$x = 0 \Rightarrow y = 9$ or $y = -9$

Answer two of the following for **Bonus** (5 pts each)

B1: Simplify the difference quotient for the function $f(x)=\sqrt{2x}$. Then pass to the limit, as h approaches zero.

~~B2: Complete the square to re-write the function~~ $\cancel{h(x)=5x^2-3x+2}$ ~~in the form~~ $\cancel{a(x-h)^2+k}$. ~~What is the vertex?~~

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[f(x+h)-f(x)\right] = \frac{1}{h}\left[\sqrt{2(x+h)}-\sqrt{2x}\right]$$

$$(a-b)\cdot(a+b) = a^2-b^2$$

$$=\frac{1}{h}\left[\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{1}\right)\left(\frac{\sqrt{2(x+h)}+\sqrt{2x}}{\sqrt{2(x+h)}+\sqrt{2x}}\right)\right]$$

$$=\frac{1}{h}\left[\frac{2(x+h)-2x}{\sqrt{2(x+h)}+\sqrt{2x}}\right] = \frac{1}{h}\left[\frac{2x+2h-2x}{\text{same}}\right] = \frac{1}{h}\left[\frac{2h}{\text{same}}\right]$$

$$=\frac{2}{\sqrt{2(x+h)}+\sqrt{2x}} \xrightarrow{h\to 0} \frac{2}{\sqrt{2x}+\sqrt{2x}} = \frac{2}{2\sqrt{2x}} = \boxed{\frac{1}{\sqrt{2x}}}$$

$h \neq 0$

B2: Complete the square to re-write the function $h(x) = 5x^2 - 3x + 2$ in the form $a(x-h)^2 + k$. What is the vertex?

$h(x) = 5x^2 - 3x + 2$

$= 5\left(x^2 - \frac{3}{5}x + \left(\frac{3}{10}\right)^2\right) + 2 - \frac{9}{10}$

$\frac{\frac{3}{5}}{2} = \frac{3}{5} \cdot \frac{1}{2} = \frac{3}{10} \rightsquigarrow \left(\frac{3}{10}\right)^2 = \frac{9}{100}$

ADDED

$5\left(\frac{9}{100}\right) = \frac{9}{20}$

$\frac{2}{1} \cdot \frac{10}{10} - \frac{9}{10} = \frac{11}{10}$

$= \boxed{5\left(x - \frac{3}{10}\right)^2 + \frac{11}{10}}$

B3: What is the domain of $r(x)=\dfrac{x-5}{x^2-5x+6}$?

If you can't factor, use quadratic formula!

$x^2-5x+6=(x-2)(x-3)=0$

$\Rightarrow x=2,3$

$\Rightarrow \mathcal{D}=\mathbb{R}\setminus\{2,3\}=\{x \mid x\neq 2 \text{ and } x\neq 3\}$

$=(-\infty,2)\cup(2,3)\cup(3,\infty)$

B4: What is the domain of $w(x)=\dfrac{x^{77}-5x^{12}+17x}{\sqrt{5-10x}}$

Need

$5-10x \geq 0$

Need

$\sqrt{5-10x} \neq 0$

Combine

$5-10x > 0$

$-10x > -5$

$\frac{-10x}{-10} < \frac{-5}{-10}$

$x < \frac{-5}{-10} = \frac{5}{10}$

$(-\infty, \frac{5}{10}) = \{x \mid x < \frac{5}{10}\}$

B5: Prove that $g(x) = -\sqrt{10-5x} + 7$ is 1-to-1.

$$\text{Pf} \quad y_2 = y_1 \implies$$

$$-\sqrt{10-5x_1} + 7 = -\sqrt{10-5x_2} + 7$$

$$+\sqrt{10-5x_1} = +\sqrt{10-5x_2}$$

$$10-5x_1 = 10-5x_2$$

$$-5x_1 = -5x_2$$

$$x_1 = x_2 \blacksquare$$

B6: Given $g(x) = -\sqrt{10-5x}+7$, find what $g^{-1}(x)$ is. $\left(\text{Hint}: (-x+7)^2 = (x-7)^2\right)$

$$-\sqrt{10-5y}+7 = x$$

$$-\sqrt{10-5y} = x-7$$

$$\sqrt{10-5y} = -(x-7)$$

$$10-5y = (x-7)^2$$

$$-5y = (x-7)^2 - 10$$

$$y = -\frac{(x-7)^2}{5} + 2 = f^{-1}(x)$$

B7: Given $g(x) = -\sqrt{10-5x} + 7$, find the domain and range of $g^{-1}(x)$.

$(-\infty, 2] = \mathcal{D}(g) = \mathcal{R}(g^{-1})$

$(-\infty, 7] = \mathcal{R}(g) = \mathcal{D}(g^{-1})$