MAT 121-G81-G82 | Test 3 – Fall, 2019 | Name Alec Granbuger
70 Points | Covers Chapter 3 | NO GRAPHING CALCULATORS!!!

Remember to use separate paper for everything except your name. Leave a margin in the top left corner. Spread out your work. Use only one column for your work. *Submit problems in order!!!*

1. (10 pts) Form a polynomial of *minimal degree* in *factored form* that has real coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong!

$x = -5$, multiplicity 3; $x = 2-7i$, multiplicity 1; $x = 2$, multiplicity 2.

$\frac{30}{70}$

AIMS Testing Center
StartTime 3:36
EndTime 4:13
Station # 17
TimeLimit 2 hours

2. (10 pts) Use synthetic division to find $P(3)$ if $P(x) = 3x^5 - 2x^4 + 17x^3 + x^2 - 10x - 4$

3. (5 pts) Represent the work you just did on the previous problem by writing $P(x)$ in the form *Dividend* = *Divisor* • *Quotient* + *Remainder* .

4. Suppose $f(x) = (x+1)^2(x-2)^3(x+4)(x-4) = x^7 - 4x^6 - 15x^5 + 74x^4 - 20x^3 - 168x^2 + 64x + 128$. I'm showing you both factored and expanded form to help you answer the following:

   a. Solve the inequality $f(x) \geq 0$. Your sign pattern for this one will be helpful in the next two. You just have to interpret what you're seeing.

   b. (10 pts) Provide a rough sketch of *f*, using its zeros, their respective multiplicities and its end behavior. Include *x*- and *y*-intercepts. Your graph should be smooth. Un-exaggerate the vertical for a better quality graph.

   c. (5 pts) What is the domain of $\sqrt{\frac{(x-2)^3(x-4)}{(x+1)^2(x+4)}}$?

5. Let $f(x) = 4x^5 - 12x^4 - 5x^3 + 21x^2 - 11x - 21$.

   a. (5 pts) Use Descartes' Rule of Signs to determine the possible number of positive and negative zeros of *f*.

   b. (5 pts) List all possible rational zeros of *f*.

   c. (Bonus 5 pts) Find the smallest possible integer bounds on positive and negative zeros.

6. (10 pts) Find the *real* zeros of $f(x) = 4x^5 - 12x^4 - 5x^3 + 21x^2 - 11x - 21$. Then factor *f* over the set of **real numbers**. This should involve an irreducible quadratic factor.

   (If things go haywire, come up with a *plausible*-looking polynomial, in factored form, with the right number of real roots and 2 nonreal roots. The 2 nonreal roots will still be living inside the irreducible quadratic

Alec Grauberger

# Test #3

1

$x = -5 \quad m^2$

$(x+5)^2$

$x = 2-7i \quad m^1$

$(x-(2-7i))(x-(2+7i))$

$x = 2 \quad m^2$

$(x-2)^2$

$m=3$

$(x+5)^2(x-(2-7i))(x-(2+7i))(x-2)^2$

9

9

(2) $P(3)$ $3x^5 - 2x^4 + 17x^3 + x^2 - 10x - 4$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3 | 3 | -2 | 17 | 1 | -10 | -4 | |
| | ↓ | 9 | 21 | 114 | 345 | 1005 | |
| | $3x^4$ | $7x^3$ | $38x^2$ | $115x$ | 335 | $\boxed{1001 = P(3)}$ | r |

10

(3) $\boxed{P(3) = (x-3)(3x^4 + 7x^3 + 38x^2 + 115 + 335) + 1001}$

$P(x)$

4

.4

5. $f(x) = 4x^5 - 12x - 5x^3 + 21x^2 - 11x - 21$

1 2 3

3 or 1

9 2 or 1 positive possible ~~numbers~~ zeros

0, based on 2.

$4(-x)^5 = -4x^5$

$f(-x) = -4x^5 + 12x + 5 - 21x^2 + 11x + 21$

2

1 possible negative ~~number~~ zeros

2 or 0

B Rational Zeros Theorem

| | | | | | | |
|---|---|---|---|---|---|---|
| -1 | 4 | -12 | -5 | 21 | -11 | -21 |
| | 4 | -4 | 16 | -11 | -10 | +21 |
| | 4 | -16 | 11 | 10 +21 | | 0 |

1

Try x = -1, again, on this

~~-2 | 4 -12 -5 21 -11 -21~~

~~-8~~

~~4 20~~

3

6 $f(x) = \overset{Q}{4x^5} - 12x^4 - 5x^3 + 21x^2 - 11x - \overset{P}{21}$

$\frac{21}{4}$

1 4 21

| | | | | | | |
|---|---|---|---|---|---|---|
| 1) | 4 | -12 | -5 | +21 | -11 | -21 |
| | ↓ | 4 | -8 | -13 | 34 | 23 |
| | 4 | -8 | -13 | 34 | 23 | +2 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 2) | 4 | -12 | -5 | 21 | -11 | -21 |
| | ↓ | 8 | -8 | -26 | -10 | -42 |
| | 4 | -4 | -13 | -5 | -21 | -63 |

| | | | | | | |
|---|---|---|---|---|---|---|
| -2) | 4 | -12 | -5 | 21 | -11 | -21 |
| | ↓ | -8 | 40 | -70 | 98 | -188 |
| | 4 | -20 | 35 | -49 | 94 | -209 |

0

6

$$\cancel{\begin{array}{r|rrrrrr} -1 & 4 & -12 & -5 & 21 & -11 & -21 \\ & & -4 & 8 & -3 & -18 & -7 \\ \hline & 4 & -8 & 3 & 18 & 7 & -44 \end{array}}$$

$$\begin{array}{r|rrrrrr} -1 & 4 & -12 & -5 & 21 & -11 & -21 \\ & \downarrow & -4 & 16 & -11 & -10 & +21 \\ \hline 1 & 4^{4} & -16^{3} & 11^{2} & 10^{1} & +21 & 0 \\ & \downarrow & 4 & -12 & 1 & 11 & \\ \hline & 4 & -12 & 1 & 11 & & \end{array}$$

Try $x=-1$, again

$$\begin{array}{r|rrrrr} 2 & 4 & -16 & 11 & 10 & 21 \\ & \downarrow & 8 & -16 & -10 & \\ \hline & 4x^2 & -8x & -5 & 0 & \end{array}$$

$4x^2-8x-5$

0

6 $\overset{a}{4x^2} \overset{b}{-8x} \overset{c}{-5}$ whence?

$-b^2 \pm -4(a)(c)$

$(-8)^2 - 4(4)(-5)$

$64 - -80$

$= 144$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-8 \pm \sqrt{144}}{8} \qquad \frac{-8i \pm 144}{8} \;\text{"}\; \frac{144}{8}$$

$= 18$

#6 – 3

#7 – 1

4

$f(0)\ 0^7 - 4(0)^6 - 15(0)^5 + 74(0) - 20(0) - 168(0) + 64(0) + 128$

$= 128$

a $4 - 15 + 74 - 20 - 168 + 64 + 128 =$

67

0

b

0