MAT 2560-R11 Written Work Chapter 1 Harry S Mills

(1) $\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}\left[c_1\sinh(kx) + c_2\cosh(kx)\right] = \frac{d}{dx}\left[kc_1\cosh(kx) + kc_2\sinh(kx)\right]$ ✓

$= k^2\sinh(kx) + k^2\cosh(kx) = \boxed{k^2\left[c_1\sinh(kx) + c_2\cosh(kx)\right]}$

$= k^2y$, where $y = c_1\sinh(kx) + c_2\cos(kx)$

We write this result as a linear 2nd order ODE w/o c_1, c_2 in the form $F(y, y', y'') = 0$:

$y'' = k^2y \Rightarrow$

$$\boxed{y'' - k^2y = 0}$$

WebAssign wants ypp $- k^2y = 0$

(2) We interpret the following statement as a differential equation

"On the graph of $y = \phi(x)$, the rate at which the slope changes w.r.t. x @ P(x,y) is the negative of the slope of the tangent line @ P(x,y).

$$\boxed{\frac{d^2y}{dx^2} = -\frac{dy}{dx} \quad \text{or} \quad y'' = -y'}$$

WebAssign: ypp = -yp.

(3) (b) we verify that the one-parameter family

(1) $y^2-2y = x^2-x+c$ is an implicit sol'n of the differential eq'n

$$(2y-2)\, y' = 2x-1.$$

Differentiating (1): $2yy'-2y' = 2x-1$

$$\Rightarrow (2y-2)y' = 2x-1 \quad \blacksquare$$

We find a member of the one-parameter family $\ni$

$y(0)=$

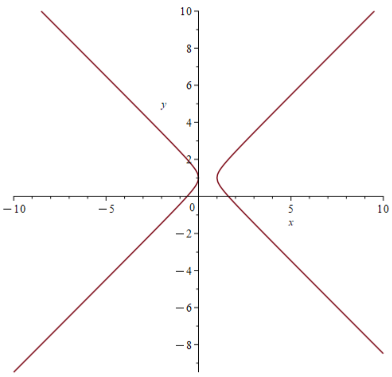
$$y(0)^2-2y(0) = 0^2-0+C \Rightarrow$$

$$1^2-2\cdot 1 = C \Rightarrow$$

$$-1 = C \Rightarrow$$

$$\boxed{y^2-2y = x^2-x-1}$$

Maple's implicitplot:



(c) We find 2 explicit sol'ns:

$$y^2-2y = x^2-x-1$$

$$y^2-2y+1^2 = x^2-x-1+1$$

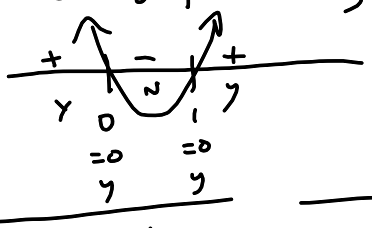
$$(y-1)^2 = x^2-x$$

$$y-1 = \pm\sqrt{x^2-x}$$

$$\boxed{y = 1 \pm \sqrt{x^2-x}} \quad \text{2 sol'ns.}$$

The domain for both sol'ns is

$$\mathcal{D} = \{x \mid x^2-x \geq 0\} = \{x \mid x(x-1) \geq 0\}$$



$$= \boxed{(-\infty, 0] \cup [1, \infty) = \mathcal{D}}$$

Note: NEITHER of the 2 sol'ns satisfies

$y(0) = 1$!

They're both sol'ns, but neither solves the IVP.

(4) We verify that $y = x\sin(x) + (\cos(x))\ln(\cos(x))$

is a sol'n of

$$y'' + y = \sec(x):$$

$$y' = \sin(x) + x\cos(x) - \sin(x)\ln(\cos(x)) + \cos(x)\left(\frac{-\sin(x)}{\cos(x)}\right)$$

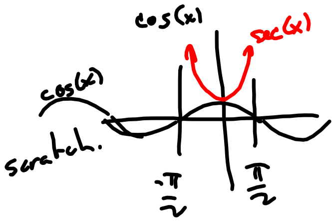
$$= \sin(x) + x\cos x - \sin(x)\ln(\cos(x)) - \sin(x)$$

$$= x\cos(x) - \sin(x)\ln(\cos(x)) \Longrightarrow$$

$$y'' = \cos(x) - x\sin(x) - \cos(x)\ln(\cos(x)) - \sin(x)\left(\frac{-\sin(x)}{\cos(x)}\right)$$

$$= \cos(x) - x\sin(x) - \cos(x)\ln(\cos(x)) + \frac{\sin^2(x)}{\cos(x)}$$

$$\Longrightarrow y'' + y = \cos(x) + \frac{\sin^2(x)}{\cos(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos(x)} = \frac{1}{\cos(x)} = \sec(x) \blacksquare$$



$\mathcal{D} = (-\frac{\pi}{2}, \frac{\pi}{2})$ is a domain that works.

So is $(\frac{\pi}{2}, \frac{3\pi}{2})$, etc.

(5) We verify that

$$y = e^{x^2}\int_0^x e^{t-t^2}\,dt \quad \text{solves}$$

$$y' - 2xy = e^x$$

pf

$$y' = 2xe^{x^2}\int_0^x e^{t-t^2}\,dt + e^{x^2}\left(e^{x-x^2}\right)$$

$$= 2xe^{x^2}\int_0^x e^{t-t^2}\,dt + e^x \Longrightarrow$$

$$y' - 2xy = 2xe^{x^2}\int_0^x e^{t-t^2}\,dt + e^x - \left(2x\cdot e^{x^2}\int_0^x e^{t-t^2}\,dt\right)$$

$$= e^x \blacksquare$$

⑥ We verify that $(x-4)^2+y^2=1$ is an implicit sol'n of $\left(\frac{dy}{dx}\right)^2+1=\frac{1}{y^2}$:

Easy for me to miscopy as $\left(\frac{d^2y}{dx}\right)$, which is entirely different.

$$\frac{d}{dx}\left[(x-4)^2+y^2=1\right] \Longrightarrow$$

$$2(x-4)+2yy'=0 \Longrightarrow$$

$$2yy'=-2x+8 \Longrightarrow$$

$$y'=\frac{-2x+8}{2y}=\frac{-2(x-4)}{2y}=-\frac{(x-4)}{y}.$$

Now, $(y')^2=\left(\frac{dy}{dx}\right)^2=\frac{(x-4)^2}{y^2}$; and by $(x-4)^2+y^2=1$, we have $(x-4)^2=1-y^2$, so that

$$\left(\frac{dy}{dx}\right)^2+1=\frac{1-y^2}{y^2}+\frac{y^2}{y^2}=\frac{1}{y^2} \blacksquare$$

(7) Given that

$y = c_1 e^{3x} + c_2 e^{-x} - 2x$ is a 2-parameter family of solutions of the 2^{nd} order (linear) ODE

$y'' - 2y' - 3y = 6x + 4$, we find c_1 & c_2 $\ni$ y satisfies the initial conditions

$y(-1) = 0,\ y'(-1) = 1$;

$y(-1) = 0 \Rightarrow c_1 e^{3(-1)} + c_2 e^{-(-1)} - 2(-1) = 0$

$\Rightarrow c e^{-3} + c_2 e + 2 = 0 \Rightarrow$

$\underline{e^{-3} c_1 + e c_2 = -2}$. Now,

$y' = 3c_1 e^{3x} - c_2 e^{-x} - 2 \Rightarrow$

$y'(-1) = 3c_1 e^{3(-1)} - c_2 e^{-(-1)} - 2$

$= 3c_1 e^{-3} - c_2 e - 2 = 1$

$\Rightarrow \underline{3e^{-3} c_1 - e c_2 = 3}$

This gives the system:

$$\begin{array}{rl} & e^{-3} c_1 + e c_2 = -2 \\ + & (3e^{-3} c_1 - e c_2 = 3) \\ \hline & 4e^{-3} c_1 = 1 \end{array}$$

$c_1 = \frac{1}{4e^{-3}} = \boxed{\frac{e^3}{4} = c_1}$

$\Rightarrow e^{-3} c_1 + e c_2 = e^{-3}\left(\frac{e^3}{4}\right) + e c_2 = e c_2 + \frac{1}{4} = -2$

$\Rightarrow e c_2 = -2 - \frac{1}{4} = \frac{-8-1}{4} = -\frac{9}{4} \Rightarrow$

$\boxed{c_2 = \frac{-9}{4e}}$

It can be proven that these are the *only* constants c1 and c2 that satisfy the initial conditions.

See Draining Tank from 8/29 Click Here

From the model for right-circular cylinder draining:

height of tank is $h_0 = 10\text{ft}$,

radius " " " $r = 3\text{ft}$

Radius Hole at bottom is $\left(\frac{3}{8}\text{in}\right)\left(\frac{1\text{ft}}{12\text{in}}\right) = \frac{1}{32}\text{ft}$. in radius.

$$\Rightarrow A_H = \pi r^2 = \pi\left(\frac{1}{32}\right)^2$$

$$\frac{dh}{dt} = -\frac{A_H}{A_w}\sqrt{2gh} = -\frac{\frac{\pi}{32^2}}{\pi(3)^2}\sqrt{2 \cdot 32 h} = \frac{-\frac{\pi}{32^2}}{9\pi} \cdot 8\sqrt{h}$$

$$= \quad \frac{dh}{dt} = -\frac{\sqrt{h}}{1152}.$$