(1) If $y = x^2 - x + 3 + 9e^x$ is a sol'n of a homogeneous 4^{th}-ORDER linear ODE w/ constant coefficients, then the roots of the C.P. are 0 with multiplicity 3 and 1 with multiplicity 1.

(2) $y = Ax^2$ is a particular sol'n for $y''' + y'' = 5$. We find A.

$y = Ax^2$
$y' = 2Ax$
$y'' = 2A$
$y''' = 0$

$\Rightarrow y''' + y'' = 0 + 2A = 5 \Rightarrow \boxed{A = \frac{5}{2}}$

(4) $4y'' + 4y' + 3y = 0 \Rightarrow$

$4r^2 + 4r + 3 = 0 \Rightarrow$

$r^2 + r + \frac{3}{4} = 0 \Rightarrow$

$r^2 + r + \left(\frac{1}{2}\right)^2 = -\frac{3}{4} + \frac{1}{4} \Rightarrow$

$\left(r + \frac{1}{2}\right)^2 = -\frac{1}{2} \Rightarrow$

$r = -\frac{1}{2} \pm \sqrt{-\frac{1}{2}} = -\frac{1 \pm i\sqrt{2}}{2} \Rightarrow$

$\boxed{y = e^{-\frac{1}{2}x}\left(c_1\cos\left(\frac{x}{\sqrt{2}}\right) + c_2\sin\left(\frac{x}{\sqrt{2}}\right)\right)}$ is the gen'l sol'n.

(5) $2y''' + 7y'' + 8y' + 3y = 0$

$2r^3 + 7r^2 + 8r + 3 = 0$

$$\begin{array}{r|rrrr} -1 & 2 & 7 & 8 & 3 \\ & & -2 & -5 & -3 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

$2r^2 + 5r + 3 = 0$

$r^2 + \frac{5}{2}r + \left(\frac{5}{4}\right)^2 = -\frac{3}{2}\cdot\frac{8}{8} + \frac{25}{16} = \frac{1}{16}$

$b^2 - 4ac$
$= 5^2 - 4(2)(3)$
$25 - 24 = 1$

$\left(r + \frac{5}{4}\right)^2 = \frac{1}{16}$

$r = \frac{-5 \pm 1}{4} = -\frac{4}{4} = -1, \quad -\frac{6}{4} = -\frac{3}{2}$

$\boxed{y = c_1 e^{-x} + c_2 x e^{-x} + c_3 e^{-\frac{3}{2}x}}$

6 $y'' - 2y' + y = x^2 e^x$

$r^2 - 2r + 1 = (r-1)^2$

$y_c = c_1 e^x + c_2 x e^x$

$y_p = (Ax^4 + Bx^3 + Cx^2 + Dx + E)e^x$

$f := x \to \exp(x) \cdot (A \cdot x^4 + B \cdot x^3 + C \cdot x^2 + D \cdot x + E)$

$$f := x \mapsto e^x \cdot (A \cdot x^4 + B \cdot x^3 + C \cdot x^2 + D \cdot x + E) \qquad (1)$$

$fp := D(f)$

$$fp := x \mapsto e^x \cdot (A \cdot x^4 + B \cdot x^3 + C \cdot x^2 + D \cdot x + E) + e^x \cdot (4 \cdot A \cdot x^3 + 3 \cdot B \cdot x^2 + 2 \cdot C \cdot x + D) \qquad (2)$$

$fpp := D(fp)$

$$fpp := x \mapsto e^x \cdot (A \cdot x^4 + B \cdot x^3 + C \cdot x^2 + D \cdot x + E) + 2 \cdot e^x \cdot (4 \cdot A \cdot x^3 + 3 \cdot B \cdot x^2 + 2 \cdot C \cdot x + D) + e^x \cdot (12 \cdot A \cdot x^2 + 6 \cdot B \cdot x + 2 \cdot C) \qquad (3)$$

$fpp(x) - 2 \cdot fp(x) + f(x)$

$$e^x (12 A x^2 + 6 B x + 2 C) \qquad (4)$$

$collect(\%, x)$

$$12 e^x A x^2 + 6 e^x B x + 2 e^x C \qquad (5)$$

$B=0, C=0$

D, E, F don't matter, so $=0$

$12Ax^2e^x = x^2e^x \Rightarrow$

$12A = 1 \Rightarrow$

$A = \frac{1}{12} \Rightarrow y_p = \frac{1}{2}x^4e^x$!

$$\boxed{y = y_c + y_p = c_1 e^x + c_2 x e^x + \frac{1}{12}x^4 e^x}$$

7 $\frac{d^2y}{dx^2} - y = \frac{2e^x}{e^x+e^{-x}} = f(x) = \frac{2}{1+e^{-2x}}$

$(D^2-1)y \Rightarrow \boxed{c_1e^x + c_2e^{-x} = y_c}$

$\begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -1 - 1 = \boxed{-2 = W(x)}$

$W_1 = \begin{vmatrix} 0 & e^{-x} \\ \frac{2e^x}{e^x+e^{-x}} & -e^{-x} \end{vmatrix} = \boxed{\frac{-2}{e^x+e^{-x}} = W_1}$

$W_2 = \begin{vmatrix} e^x & 0 \\ e^x & \frac{2e^x}{e^x+e^{-x}} \end{vmatrix} = \boxed{\frac{2e^{2x}}{e^x+e^{-x}} = W_2}$

$y_p = u_1y_1 + u_2y_2 = y_1\int \frac{W_1}{W}dx + y_2\int \frac{W_2}{W}dx$

$= y_1\int -\frac{1}{2}\left(\frac{-2}{e^x+e^{-x}}\right)dx + y_2\int -\frac{1}{2}\left(\frac{2e^{2x}}{e^x+e^{-x}}\right)dx$

$= e^x\int \frac{dx}{e^{-x}(e^{2x}+1)} - e^{-x}\int \frac{e^{2x}}{e^{-x}(e^{2x}+1)}dx$

$= e^x \arctan(e^x) - \frac{1}{2}e^{-x}\int \frac{e^{3x}}{e^{2x}+1}dx$

$$\begin{array}{r} e^x \\ e^{2x}+1 \overline{\big) e^{3x}} \\ -(e^{3x}+e^x) \\ \hline -e^x \end{array}$$

$= e^x \arctan(e^x) - \frac{1}{2}e^{-x}\int\left(e^x - \frac{e^x}{e^{2x}+1}\right)dx$

$= \boxed{e^x\arctan(e^x) - 1 + e^x\arctan(e^x)}$

(8) $x^2y'' - 9xy' + 25y = x^3$

is of Cauchy-Euler type.

$a(m(m-1)) + bm + c = am^2 + (b-a)m + c$

$a=1,\ b=-9,\ c=25$

$(b-a)^2 - 4ac = (-9-1)^2 - 4(1)(25)$

$= 10^2 - 100 = 0$

repeated roots.

$m = \frac{a-b}{2} = \frac{1-(-9)}{2} = 5$

So $\boxed{y_h = c_1x^5 + c_2x^5\ln(x)}$

y_p: $y'' - \frac{9}{x}y' + \frac{25}{x^2} = x$

$$w = \begin{vmatrix} x^5 & x^5\ln(x) \\ 5x^4 & 5x^4\ln(x) + x^4 \end{vmatrix} = 5x^9\ln x + x^9 - 5x^9\ln(x) = x^9$$

$$w_1 = \begin{vmatrix} 0 & x^5\ln(x) \\ x & 5x^4\ln(x) + x^4 \end{vmatrix} = -x^6\ln(x)$$

$$w_2 = \begin{vmatrix} x^5 & 0 \\ 5x^4 & x \end{vmatrix} = x^6$$

$$y_p = x^5\int \frac{w_1}{w}\,dx + x^5\ln(x)\int \frac{w_2}{w}\,dx$$

$$= x^5\int \frac{-x^6\ln(x)}{x^9}\,dx + x^5\ln(x)\int \frac{x^6}{x^9}\,dx$$

$$= -x^5\int \frac{\ln(x)}{x^3}\,dx + x^5\ln(x)\int x^{-3}\,dx$$

$u = \ln(x)$ $\quad dv = \frac{1}{x^3}\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = -\frac{1}{2}x^{-2}$

$$x^5\left(uv - \int v\,du\right) + x^5\ln(x)\left[-\frac{1}{2}x^{-2}\right]$$

$$= -x^5\left(\ln(x)\left(-\frac{1}{2}x^{-2}\right)\right) - \int\left(\frac{1}{2}x^{-2}\right)\left(\frac{1}{x}\,dx\right)\Big) - \frac{1}{2}x^3\ln(x)$$

$$\frac{1}{2}x^3\ln(x) - \frac{x^5}{2}\int x^{-3}\,dx - \frac{1}{2}x^3\ln(x)$$

$$= -\frac{x^5}{2}\left(\frac{x^{-2}}{-2}\right) = \boxed{\frac{x^3}{4} = y_p}$$

must be an easier way! Shouldn't tried Ax^3+Bx^2+Cx+D?

9 $y'' - \omega^2 y = e^{\alpha x}$

i. $\alpha \neq \omega$

ii. $\alpha = \omega$

i. $r^2 - \omega^2 = (r-\omega)(r+\omega) \Rightarrow$

$\boxed{y_h = c_1 e^{\omega x} + c_2 e^{-\omega x}}$. $\alpha \neq \omega$

y_p: $y_1 = e^{\omega x}$, $y_2 = e^{-\omega x}$

$$W = \begin{vmatrix} e^{\omega x} & e^{-\omega x} \\ \omega e^{\omega x} & -\omega e^{-\omega x} \end{vmatrix} = e^{\omega x}(-\omega e^{-\omega x}) - \omega e^{\omega x} e^{-\omega x} = \boxed{-2\omega = W}$$

$$W_1 = \begin{vmatrix} 0 & e^{-\omega x} \\ e^{\alpha x} & -\omega e^{-\omega x} \end{vmatrix} = -e^{\alpha x - \omega x} = \boxed{-e^{(\alpha-\omega)x} = W_1}$$

$$W_2 = \begin{vmatrix} e^{\omega x} & 0 \\ \omega e^{\omega x} & e^{\alpha x} \end{vmatrix} = \boxed{e^{(\alpha+\omega)x} = W_2}$$

$$y_p = e^{\omega x}\int \frac{W_1}{W}\,dx + e^{-\omega x}\int \frac{W_2}{W}\,dx$$

$$= e^{\omega x}\int \frac{-e^{(\alpha-\omega)x}}{-\omega} + e^{-\omega x}\int \frac{e^{(\alpha+\omega)x}}{-\omega}\,dx =$$

$$= \frac{1}{\omega}e^{\omega x}\int e^{(\alpha-\omega)x}\,dx - \frac{1}{\omega}e^{-\omega x}\int e^{(\alpha+\omega)x}\,dx$$

$$= \frac{e^{\omega x}}{\omega(\alpha-\omega)}e^{(\alpha-\omega)x} - \frac{e^{-\omega x}}{\omega(\alpha+\omega)}e^{(\alpha+\omega)x}$$

$$= \frac{e^{\alpha x}}{\omega(\alpha-\omega)} - \frac{e^{\alpha x}}{\omega(\alpha+\omega)} = \frac{(\alpha+\omega)e^{\alpha x} - (\alpha-\omega)e^{\alpha x}}{\omega(\alpha-\omega)(\alpha+\omega)}$$

$$= \frac{2\omega e^{\alpha x}}{\omega(\alpha-\omega)(\alpha+\omega)} = \frac{2e^{\alpha x}}{(\alpha-\omega)(\alpha+\omega)}$$

$$= \boxed{Ae^{\alpha x} = y_p(x)}$$

(ii) $\alpha = \omega$

$$\boxed{\begin{aligned} W &= -2\omega \\ W_1 &= -1 \\ W_2 &= e^{2\alpha x} \end{aligned}}$$

$$y_p = e^{\omega x}\int \frac{W_1}{W}\,dx + e^{-\omega x}\int \frac{W_2}{W}\,dx$$

$$= e^{\omega x}\int \frac{-1}{-2\omega}\,dx + e^{-\omega x}\int \frac{e^{2\omega x}}{-2\omega}\,dx$$

$$= \frac{1}{2\omega} e^{\omega x}\cdot x - \frac{1}{2\omega} e^{-\omega x}\left(\frac{1}{2\omega} e^{2\omega x}\right)$$

WebAssign doesn't want this detail.

$$= \frac{1}{2\omega} x e^{\omega x} - \frac{1}{4\omega^2} e^{\omega x} = y_p$$

Part of y_h. Absorb the constant $\frac{1}{4\omega^2}$ into $\hat{c} = c_1 - \frac{1}{4\omega^2}$. Then

$$y = y_h + y_p = c_1 e^{\omega x} + c_2 e^{-\omega x} + \frac{1}{2\omega} x e^{\omega x} - \frac{1}{4\omega^2} e^{\omega x}$$

$$\boxed{y = \hat{c} e^{\omega x} + c_2 e^{-\omega x} + A x e^{\omega x}}$$

WebAssign doesn't want A to be given in terms of ω.