$y(\ln(x) - \ln(y))\,dx = (x\ln(x) - x\ln(y) - y)\,dy$

Hint: combine logs. Let $\frac{y}{x} = v$, $y = vx$

$$y\left(\ln\left(\tfrac{x}{y}\right)\right)dx = \left(x\ln\left(\tfrac{x}{y}\right) - y\right)dy$$
$$-y\,\ln\left(\tfrac{y}{x}\right)dx = -\left(x\ln\left(\tfrac{y}{x}\right) + y\right)dy$$
$$y\ln\left(\tfrac{y}{x}\right)dx = \left(x\ln\left(\tfrac{y}{x}\right) + y\right)dy$$

Let $v = \frac{y}{x}$. Then $y = vx$ & $dy = (v'x + v)\,dx = (xv' + v)\,dx$
$= x\,dv + v\,dx$ and so

$$vx\ln(v)\,dx = (x\ln(v) + vx)(x\,dv + v\,dx)$$
$$= x^2\ln(v)\,dv + xv\ln(v)\,dx + x^2v\,dv + xv^2\,dx$$
$$0 = x^2\ln(v)\,dv + x^2v\,dv + xv^2\,dx$$
$$-xv^2\,dx = x^2(\ln(v) + v)\,dv$$

Separable!

$$-\frac{1}{x}\,dx = \frac{\ln(v) + v}{v^2}\,dv$$
$$-\ln|x| = \int \ln(v)\left(\frac{1}{v^2}\,dv\right) + \int \frac{1}{v}\,dv = \int \frac{\ln(v)}{v^2}\,dv + \ln(v) + C$$
$$= \ln(v) + \int \frac{\ln(v)}{v^2}\,dv = \ln(v) + \ln(v)\left(-\frac{1}{v}\right) - \int -\frac{1}{v}\cdot\frac{1}{v}\,dv$$

$\int u\,dg = ug - \int g\,du$

$u = \ln(v)$, $du = \frac{1}{v}\,dv$, $dg = \frac{1}{v^2}$, $g = -\frac{1}{v}$

$$= \ln(v) - \frac{1}{v}\ln(v) + \int \frac{1}{v^2}\,dv$$
$$= \ln(v) - \frac{1}{v}\ln(v) - \frac{1}{v} + C$$

$$-\ln(x) = \ln\left(\tfrac{y}{x}\right) - \frac{1}{\frac{y}{x}}\ln\left(\tfrac{y}{x}\right) - \frac{x}{y} + C$$

$$\boxed{-\ln(x) + C_1 = \ln\left(\tfrac{y}{x}\right) - \frac{x}{y}\ln\left(\tfrac{y}{x}\right) - \frac{x}{y}}$$

2 $\frac{dx}{dy} = -\frac{2y^2+14xy}{7y^2+2x}$

$(7y^2+2x)\,dx + (2y^2+14xy)\,dy = 0$

$M_y = 14y \quad N_x = 14y \Rightarrow$ EXACT

$f(x,y) = \int M\,dx + g(y)$

$f_y = \frac{d}{dy}\int M\,dx + g'(y) = \frac{d}{dy}\left[\int (7y^2+2x)\,dx\right] + g'(y)$

$= \frac{d}{dy}\left[7y^2x + x^2\right] + g'(y)$

$= 14yx + g'(y) = N = 2y^2+4xy$

$\Rightarrow g'(y) = 2y^2 \Rightarrow g(y) = \frac{2}{3}y^3$

$\Rightarrow f(x,y) = \boxed{7xy^2 + x^2 + \frac{2}{3}y^3 = C} = $ sol'n

3. $(2x+y+1)y' = 1$

Let $u = 2x+y+1 \longrightarrow$

$\frac{du}{dx} = 2 + \frac{dy}{dx} \longrightarrow \frac{dy}{dx} = \frac{du}{dx} - 2$

$\Longrightarrow u\left(\frac{du}{dx} - 2\right) = 1$

$\Longrightarrow \frac{du}{dx} - 2 = \frac{1}{u}$

$\frac{du}{dx} = \frac{1}{u} + 2 = \frac{1+2u}{u}$

$\frac{u\,du}{2u+1} = dx$

$v = 2u+1 \Longrightarrow \frac{v-1}{2} = u$

$dv = 2du$

$du = \frac{dv}{2}$

$\int \frac{u\,du}{2u+1} = \int \frac{\frac{v-1}{2}}{v} \cdot \frac{dv}{2} = \frac{1}{4}\int \frac{v-1}{v}\,dv = \frac{1}{4}\int\left(1 - \frac{1}{v}\right)dv = \frac{1}{4}v - \frac{1}{4}\ln|v| = x + C$

$\Longrightarrow \frac{1}{4}(2u+1) - \frac{1}{4}\ln|2u+1| = x + C$

$\Longrightarrow \frac{1}{4}(2(2x+y+1)+1) - \frac{1}{4}|2(2x+y+1)+1| = x + C$

$\Longrightarrow \frac{1}{4}(4x+2y+2+1) - \frac{1}{4}\ln|4x+2y+2+1| = x + C$

$\Longrightarrow \underline{x} + \frac{1}{2}y + \frac{3}{4} - \frac{1}{4}\ln|4x+2y+3| = \underline{x} + C$

$\Longrightarrow 2y + 3 - \ln|4x+2y+3| = 4C$

Absorb into C

$$\boxed{\Longrightarrow 2y - \ln|4x+2y+3| = \hat{C}}$$

(4) $(2r^2\cos\theta\sin\theta + r\cos\theta)\,d\theta + (4r + \sin\theta - 2r\cos^2\theta)\,dr = 0$

$M_r = 4r\cos\theta\sin\theta + \cos\theta \qquad N_\theta = \cos\theta + 4r\sin\theta\cos\theta \longrightarrow$ EXACT

N is easier to integrate wrt 'r' than M wrt 'θ.'

$f(r,\theta) = \int N\,dr + g(\theta) \longrightarrow$

$f_\theta = \frac{\partial}{\partial\theta}\int (4r + \sin\theta - 2r\cos^2\theta)\,dr + g'(\theta)$

$= \frac{\partial}{\partial\theta}\left[2r^2 + r\sin\theta - r^2\cos^2\theta\right] + g'(\theta)$

$= r\cos\theta + 2r^2\sin\theta\cos\theta + g'(\theta) = M = 2r^2\sin\theta\cos\theta + r\cos\theta$

$\longrightarrow g'(\theta) = 0 \longrightarrow g(\theta) = C \overset{\text{LET}}{=} 0$

$\longrightarrow f(r,\theta) = \boxed{2r^2 + r\sin\theta - r^2\cos^2\theta = \hat{C}}$ is implicit soln.

⑤ $\frac{dy}{dx} - 4xy = \sin(x^2)$, s.t. $y(0)=9$ is our IVP.

μ = integrating factor $= e^{-\int 4x\,dx} = e^{-4\cdot\frac{x^2}{2}+c} = Ke^{-2x^2} = \boxed{e^{-2x^2} = \mu(x)}$

$K = e^c$ $\quad$ $K=1$

$\mu y' - 4x\mu y = \mu \sin(x^2)$

$= (\mu y)' = \mu \sin(x^2) \longrightarrow$

$\mu y = \int \mu \sin(x^2)\,dx = \int e^{-2x^2} \sin(x^2)\,dx + C$

$y(0) = 9 \longrightarrow$

$\mu(x)\,y(x) - \mu(0)\,y(0) = \int_0^x e^{-2t^2} \sin(t^2)\,dt$

$e^{-2x^2} y(x) - e^{-2(0)^2}(9) = e^{-2x^2} y(x) - 9 = \int_0^x g(t)\,dt$, where

$\boxed{g(t) = e^{-2t^2} \sin(t^2)}$. $\longrightarrow$

$e^{-2x^2} y(x) = \int_0^x g(t)\,dt + 9 \longrightarrow$

$$\boxed{y(x) = e^{2x^2} \int_0^x g(t)\,dt + 9e^{-2x^2}}$$

(6) $y' + P(x)y = e^x$ s.t. $y(0) = -2$ is our IVP.

Given $P(x) = \begin{cases} 1 & \text{if } 0 \le x < 1 \\ -1 & \text{if } 1 \le x \end{cases}$, we find the soln on $\mathbb{R}$.

$x \in [0,1)$

$y' + y = e^x$

$\mu = e^{\int dx} = e^{x+c} = Ke^x = e^x$, via $K = e^c = 1$, for convenience

$(ye^x)' = e^{2x}$

$ye^x = \int e^{2x}dx = \frac{1}{2}e^{2x} + C$

$y(0) = -2 \longrightarrow$

$-2 = \frac{1}{2} + C \longrightarrow \boxed{C = -\frac{5}{2}}$

$\Longrightarrow y = e^{-x}\left(\frac{1}{2}e^{2x} - \frac{5}{2}\right)$

$\Longrightarrow \boxed{y = \frac{1}{2}e^x - \frac{5}{2}e^{-x} \text{ on } [0,1)}$

$x \in [1,\infty)$:

$P(x) = -1$: $\mu = e^{\int P(x)dx} = Ke^{-x} = \boxed{e^{-x} = \mu(x)}$

$(\mu y)' = e^{-x}e^x = 1 \longrightarrow$

$\mu y = x + C$

$y = \frac{1}{\mu}x + \frac{C}{\mu} = xe^x + ce^x$

$\lim_{x\to 1^-} y(x) = \frac{1}{2}e^1 - \frac{5}{2}e^{-1} = \frac{1}{2}e - \frac{5}{2e} = y(1)$, $\therefore$ y is cts

$= \frac{1}{2}e(1 - 5e^{-2})$

$\Longrightarrow$ on $[1,\infty)$, we have

$y(1) \equiv e^1 + ce^1 = e(c+1) = \frac{1}{2}e(1 - 5e^{-2})$

$\longrightarrow c + 1 = \frac{1}{2} - \frac{5}{2}e^{-2} \longrightarrow$

$c = -\frac{1}{2} - \frac{5e^{-2}}{2}$

$y = xe^x + \left(\frac{1}{2} - \frac{5e^{-2}}{2}\right)e^x$

$$\boxed{y = xe^x - \frac{1}{2}e^{x} - \frac{5e^{x-2}}{2}}$$

(7) We solve the IVP $\frac{dy}{dt}+2(t+1)y^2=0$ s.t $y(0)=-\frac{1}{3}$

separable:

$$\frac{dy}{dt} = -2(t+1)y^2 \to$$

$$\frac{dy}{y^2} = -2\int(t+1)\,dt = -2\left[\tfrac{1}{2}t^2+t\right]+C$$

$$-y^{-1} = -t^2-2t+C$$

$$y(0)=-\tfrac{1}{3} \to$$

$$-(-3) = C \Rightarrow \boxed{C=3}$$

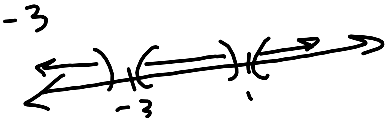
$$\Rightarrow -\frac{1}{y} = -t^2-2t+3$$

$$\Rightarrow \boxed{y=\frac{1}{t^2+2t-3}}$$

Largest integral of definition is the interval containing $t=0$ that works.

Need $t^2+2t-3=(t+3)(t-1)\neq 0 \to$

$t\neq 1,\ t\neq -3$

This gives 

$t=0\in(-3,1) \to \boxed{(-3,1)}$

8 Euler's for $y' = 2 + x\sqrt{y}$, s.t. $y(1) = 25$.

Use step size $h = 0.1$

$$y(x_{n+1}) \approx y'(x_n)h + y(x_n)$$

$$= (2 + x_n y(x_n))h + y(x_n)$$

h 2 Written	$y' = 2 + x\sqrt{y}$; $y(1) = 25$				
Vork #8				We estimate y(1.2):	
	h = 0.1				
	h	*n*	*xn*	*yn*	
	0.1	0	1	25	
		1	1.1	25.7	
		2	1.2	26.4576	
		3	1.3	27.2749	
		4	1.4	28.1538	
		5	1.5	29.0967	

	A	B	C	D	E
1					
2	Ch 2 Written		$y' = 2 + x\sqrt{y}$; $y(1) = 25$		
3	Work #8				We estimate y(1.2):
4		**h = 0.1**			
5		*h*	*n*	*xn*	*yn*
6		0.1	0	1	25
7			=C6+1	=D6+$B$6	=$B$6*(2+D6*SQRT(E6))+E6
8			=C7+1	=D7+$B$6	=$B$6*(2+D7*SQRT(E7))+E7
9			=C8+1	=D8+$B$6	=$B$6*(2+D8*SQRT(E8))+E8
10			=C9+1	=D9+$B$6	=$B$6*(2+D9*SQRT(E9))+E9