MAT 2560-R11 Final Test, Fall, 2024 Name_____
100 points Covers Chapters 4 - 7

Use the blank pages provided to show all your work and final answers. **Circle Final Answers**.

Leave room before, after, and within your work.

Do not use a separate scratch sheet. I need to see your scratch work and I need to see it next to (within) the problem it supports. That's why I give you blank paper. Write as much as you want, as big as you want.

1. (10 pts) If $y = 3x^2 - 5x + 2 - 7e^{-3x} + 4xe^{-3x}$ is a solution of a homogeneous 4th-order linear differential equation with constant coefficients, then describe the roots of the auxiliary equations: What are they and what are their respective multiplicities?

2. Solve the homogeneous linear ODE $y'' - 9y = 0$, subject to the initial conditions $y(0) = 8, y'(0) = -6$ in two ways:

   a. (10 pts) Find the roots of the auxiliary equation and use them to write the solution as a linear combination of exponential functions e^{at} and e^{bt}.

   b. (10 pts) Use the power series method of Frobenius. Remember that the initial conditions give you the first two terms. I want to see 5 terms.

BONUS – (10 pts) Check your work for #2 by computing the first 5 terms of the power series expansion for your answer to #2***a***.

3. We solve $y'' - 9y = 8e^t$, subject to the initial conditions $y(0) = 7, y'(0) = -7$ in various ways:

   a. (10 pts) Find the particular solution by method of undetermined coefficients. Then use the principle of superposition and your solution to the homogeneous equation in #2***a***. to construct the solution of the nonhomogeneous IVP. Finally, incorporate the initial conditions into your answer.

   b. (10 pts) Find the particular solution by the method of variation of parameters.

   c. (10 pts) Use a Green's function to write the solution as one integral. Why would you ever want to go to the extra work of a Green's-function solution?

   d. (10 pts) Use Laplace Transforms to clobber the whole thing at one go. I'll give you a hint:

$$\frac{7s^2 - 14s + 15}{(s-1)(s-3)(s+3)} = \frac{5}{s+3} + \frac{3}{s-3} - \frac{1}{s-1}$$

   e. (10 pts) Use the method of Frobenius to solve the nonhomogeneous equation $y'' - 9y = 8e^t$ subject to the initial conditions $y(0) = 7, y'(0) = -7$, that is, give a power series solution.

4. (10 pts) Recall that Kirchoff's second law says that the sum of the voltage drops in an LRC circuit is equal to the total voltage drop, which is to say:

   $L\frac{di}{dt}+Ri+\frac{1}{C}\int_0^t i(\tau)\,d\tau = E(t)$, where L is the inductance in Henries, R is the resistance in ohms, C is the capacitance in farads. Given that L = ½ h, R = 10 Ω, C = 1/100 f, and $E(t)=10$ volts, solve for the current $i(t)$, given in amperes. Then solve for the charge q, given that $\frac{dq}{dt}=i$, and $q(0)=0$

5. (10 pts) Decompose $\frac{1}{(s-1)(s-3)(s+3)}$ by partial fractions.