4.1 #11

Verify that the given two-parameter family of functions is the general solution of the nonhomogeneous differential equation on the indicated interval.

$2x^2y'' + 5xy' + y = x^2 - x;$

$y = c_1x^{-1/2} + c_2x^{-1} + \frac{1}{15}x^2 - \frac{1}{6}x,$ $(0, \infty)$ $\Rightarrow$ Pay Attention!

The functions $x^{-1/2}$ and x^{-1} satisfy the differential equation and are linearly independent since

$W(x^{-1/2}, x^{-1}) =$ $-\frac{1}{2x^{5/2}}$ $\neq 0$ for $0 < x < \infty$. So the functions $x^{-1/2}$ and x^{-1} form a fundamental set

of solutions of the associated homogeneous equation, and $y_p =$ $y_p = \frac{1}{15}x^2 - \frac{1}{6}x$ is a particular solution of

the nonhomogeneous equation.

Check: $y = c_1x^{-\frac{1}{2}} + c_2x^{-1}$

$\Rightarrow y' = -\frac{1}{2}x^{-\frac{3}{2}} - c_2x^{-2}$

$y'' = \frac{3}{4}x^{-\frac{5}{2}} + 2c_2x^{-3}$

Right-click and download Maple work for confirming they are solutions.

See PDF of the Maple work.

y_p: Assume sol'n is of the form $c_1x^2 + c_2x = y$

$y' = 2c_1x + c_2$

$y'' = 2c_1$

$2x^2y'' + 5xy' + y$

$2x^2(2c_1) + 5x(2c_1x + c_2) + c_1x^2 + c_2x = x^2 - x$

Find c_1, c_2: $\Rightarrow$

x^2: $(4c_1 + 10c_1 + c_1)x^2 = x^2 \Rightarrow \overset{15}{7}c_1x^2 = x^2 \Rightarrow \overset{15}{7}c_1 = 1 \Rightarrow c_1 = \frac{1}{7}$

$c_1 = \frac{1}{15}$

x: $(5c_2 + c_2)x = -x \Rightarrow 6c_2 = -1 \Rightarrow c_2 = -\frac{1}{6} \Rightarrow$

$$\boxed{y_p = \frac{1}{15}x^2 - \frac{1}{6}x}$$

$2x^2(2c_1) + 5x(2c_1x + c_2) + c_1x^2 + c_2x = x^2 - x \Rightarrow$

$4c_1x^2 + 10c_1x^2 + 5c_2x + c_1x^2 + c_2x = x^2 - x \Rightarrow$

$4c_1x^2 + 10c_1x^2 + c_1x^2 = x^2$ &

$5c_2x + c_2x = -x$

would've / could've eliminated some mistakes.

Write much. Think little.