Section 4.1 #4

Consider the following functions.

$$f_1(x) = x, \quad f_2(x) = x^2, \quad f_3(x) = 6x - 4x^2$$
$$g(x) = c_1f_1(x) + c_2f_2(x) + c_3f_3(x)$$

Solve for c_1, c_2, and c_3 so that $g(x) = 0$ on the interval $(-\infty, \infty)$. If a nontrivial solution exists, state it. (If only the trivial solution exists, enter the trivial solution $\{0, 0, 0\}$.)

$\{c_1, c_2, c_3\} = \{ \quad \}$

Determine whether f_1, f_2, f_3 are linearly independent on the interval $(-\infty, \infty)$.

M1 $W(f_1, f_2, f_3)$

$$\begin{vmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{vmatrix} = \begin{array}{c} + \\ - \\ + \end{array}\begin{vmatrix} x & x^2 & 6x-4x^2 \\ 1 & 2x & 6-8x \\ 0 & 2 & -8 \end{vmatrix} = 0!$$

Nice

$= x(-16x - (12-16x)) - 1(-8x^2 - (12x - 8x^2)) + 0(\sim)$

$= x(-16x - 12 + 16x) - 1(-8x^2 - 12x + 8x^2)$

$= -12x + 12x = 0$! Dependent!

So we need to find the dependence relationship

$c_1 x + c_2 x^2 + c_3(6x - 4x^2)$

Let $c_1 = -6c_3$

$c_2 = +4c_3$

Let $c_3 = 1$

$c_1 = -6$

$c_2 = 4$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 4 \\ 1 \end{bmatrix}$$ By inspection.

$c_1 x + c_2 x^2 + c_3(6x - 4x^2) \overset{\text{SET}}{=} 0 \Longrightarrow$

$c_1 x + c_2 x^2 + 6c_3 x - 4c_3 x^2 = 0 \Longrightarrow$

$c_1 + 6c_3 = 0$ &

$c_2 - 4c_3 = 0$

$\Longrightarrow$

$c_1 \quad + 6c_3 = 0$

$c_2 - 4c_3 = 0$

$$\left[\begin{array}{ccc|c} 1 & 0 & 6 & 0 \\ 0 & 1 & -4 & 0 \end{array}\right] \Longrightarrow$$

$c_1 = -6c_3$

$c_2 = 4c_3$

c_3 = ANY.

c_3 is "free"

Reduced Row-Echelon Form

$$\begin{array}{cccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \\ & & & & & & 0 \\ 1 & 0 & 0 & 0 & 1 & 5 & 0 \\ 0 & 1 & 0 & 0 & -5 & 2 & 0 \\ 0 & 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & 3 & 1 & 0 \end{array}$$

x_1, x_2, x_3, x_4 depend on free variables x_5, x_6.

$$Wronskian([x, x^2, 6\cdot x - 4\cdot x^2], x, 'determinant')$$

$$\left[\begin{array}{ccc} x & x^2 & -4x^2 + 6x \\ 1 & 2x & -8x + 6 \\ 0 & 2 & -8 \end{array}\right], 0$$

$W(x, x^2, 6x - 4x^2)$