

$$y = c_1 x + c_2 x^4$$

$$y' = c_1 + 4c_2 x^3$$

$$y'' = 12c_2 x^2$$

$$a(x)y'' + b(x)y' + c(x)y = 0$$

$$12c_2 x^2 a(x) + (c_1 + 4c_2 x^3)b(x) + (c_1 x + c_2 x^4)c(x) = 0$$

c_1 's must go away:

$$c_1 b(x) + c_1 x c(x) = 0 \rightarrow$$

$$b(x) = -x c(x)$$

c_2 's must go away if $b(x) = -x c(x) \rightarrow$

$$12c_2 x^2 a(x) + 4c_2 x^3 b(x) + c_2 x^4 c(x)$$

$$= 12c_2 x^2 a(x) + 4c_2 x^3 (-x c(x)) + c_2 x^4 c(x) = 0 \rightarrow$$

$$12x^2 a(x) - 4x^4 c(x) + x^4 c(x)$$

$$= 12x^2 a(x) - 3x^4 c(x) = 0$$

$$\Rightarrow 12x^2 a(x) = 3x^4 c(x)$$

$$\Rightarrow a(x) = \frac{3x^4 c(x)}{12x^2} = \frac{1}{4} x^2 c(x)$$

Let $c(x) = 4$. Then

$$a(x) = \frac{1}{4} x^2 c(x) = \frac{1}{4} x^2 (4) = x^2 = a(x)$$

$$\text{if } b(x) = -x(4) = -4x.$$

$$\text{Then } a(x)y'' + b(x)y' + c(x)y$$

$$= x^2 y'' - 4xy' + 4y$$

$$= x^2 y'' - 4xy' + 4y = 0$$