10 $E(t) = \begin{cases} 10 & 0 \le t < 5 \\ 0 & t \ge 5 \end{cases}$ is the voltage driving the circuit.

We have an LRC circuit where inductance $= L = \frac{1}{2}$ henry, $R = 10\,\Omega$ (ohms, i.e.), & $C = 0.01$ Farad.

Kirchoff says:

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int_0^t i(\tau)\,d\tau = E(t)$$

where $i =$ the current, in amps

$$\frac{1}{2}i' + 10i + \frac{1}{.01}\int_0^t i(\tau)\,d\tau = 10\,\mathcal{U}(t) - 10\,\mathcal{U}(t-5)$$

Find $q(t) = \int_0^t i(\tau)\,d\tau$, given $q(0) = q'(0) = 0$.

(i) Solve for $i(t)$

(ii) Integrate to $q(t)$.

Recall: $i(t) =$ current $= \frac{dq}{dt} =$ rate of change in charge w.r.t. time.

(i) $\mathcal{U}(t) =$ Heaviside(t) in Maple.

#10

I'm getting lazy. There's nothing new here, except the level of difficulty.

$with(inttrans)$

$[addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable, setup]$

$unassign('y(t)')$ ← Didn't use

$L := \frac{1}{2}; R := 10; C := \frac{1}{100};$

$$L := \frac{1}{2}$$

$$R := 10$$

$$C := \frac{1}{100}$$

Using $x(t)$ for $i(t)$

$eqn := L\cdot diff(x(t), t) + R\cdot x(t) + \frac{1}{C}\cdot\int_0^t x(\mathrm{tau})\,\mathrm{dtau} = 10 - 10\cdot\mathrm{Heaviside}(t-5)$

$$eqn := \frac{\frac{d}{dt}x(t)}{2} + 10\,x(t) + 100\left(\int_0^t x(\tau)\,d\tau\right) = 10 - 10\,\underbrace{\mathrm{Heaviside}(t-5)}_{\mathcal{U}(t-5)}$$

$convert(\mathrm{Heaviside}(t-1), piecewise)$

$$\begin{cases} 0 & t < 1 \\ 1 & 1 \le t \end{cases}$$

To make Heaviside function = 1 at 0:

$NumericEventHandler(invalid\_operation = `Heaviside/EventHandler`(value\_at\_zero = 1)):$

$laplace(eqn, t, s)$

$$\frac{s\,\mathcal{L}(x(t), t, s)}{2} - \frac{x(0)}{2} + 10\,\mathcal{L}(x(t), t, s) + \frac{100\,\mathcal{L}(x(t), t, s)}{s} = \frac{10\left(1 - e^{-5s}\right)}{s}$$

$solve(\%, laplace(x(t), t, s))$

$$-\frac{-x(0)\,s + 20\,e^{-5s} - 20}{s^2 + 20\,s + 200}$$

$subs(x(0) = 0, \%)$

$$-\frac{-20 + 20\,e^{-5s}}{s^2 + 20\,s + 200}$$

If $g(0) = g'(0) = 0$, it's reasonable to think $i(0) = i'(0) = 0$.

$invlaplace(\%, s, t)$

$$2\,e^{-10t}\sin(10\,t) - 2\,\mathrm{Heaviside}(t-5)\,e^{-10t+50}\sin(10\,t-50)$$

$g'(0)$ we flip the switch at $t=0$.

$convert(\%, piecewise)$

$$\begin{cases} 2\,e^{-10t}\sin(10\,t) & t < 5 \\ 2\,e^{-10t}\sin(10\,t) - 2\,e^{-10t+50}\sin(10\,t-50) & 5 \le t \end{cases}$$

$simplify(\%)$

$$2\,e^{-10t}\sin(10\,t) - \left(\begin{cases} 0 & t < 5 \\ 2\,e^{-10t+50}\sin(10\,t-50) & 5 \le t \end{cases}\right)$$

Integrate to the solution q:

$$\int_0^t 2\,\mathrm{e}^{-10\,\mathrm{tau}}\sin(10\,\mathrm{tau}) - \left(\begin{cases} 0 & \mathrm{tau} < 5 \\ 2\,\mathrm{e}^{-10\,\mathrm{tau}+50}\sin(10\,\mathrm{tau}-50) & 5 \le \mathrm{tau} \end{cases}\right)\mathrm{dtau}$$

$$\frac{1}{10} - \frac{\mathrm{e}^{-10\,t}\cos(10\,t)}{10} - \frac{\mathrm{e}^{-10\,t}\sin(10\,t)}{10} - \left(\begin{cases} 0 & t \le 5 \\ -\frac{\mathrm{e}^{-10\,t+50}\cos(10\,t-50)}{10} - \frac{\mathrm{e}^{-10\,t+50}\sin(10\,t-50)}{10} + \frac{1}{10} & 5 < t \end{cases}\right)$$

$convert(\%, \mathrm{Heaviside})$

$$\frac{1}{10} - \frac{\mathrm{e}^{-10\,t}\cos(10\,t)}{10} - \frac{\mathrm{e}^{-10\,t}\sin(10\,t)}{10} + \frac{\mathrm{Heaviside}(t-5)\,\mathrm{e}^{-10\,t+50}\cos(10\,t-50)}{10} + \frac{\mathrm{Heaviside}(t-5)\,\mathrm{e}^{-10\,t+50}\sin(10\,t-50)}{10} - \frac{\mathrm{Heaviside}(t-5)}{10}$$