9 Solve

$$y_1'' + y_2'' = e^{2t}$$
$$y_1' + y_2'' = -e^{2t}$$

$y_1(0) = y_1'(0) = y_2(0) = y_2'(0) = 0$

I miscopied the coefficient of y1' in the 2nd equation. Eventually, I got tired of trying to solve it, regardless. I'm probably missing a shortcut that makes it easier to do by hand. I don't have to think very hard if I see the big picture and have Maple, though. See Maple links, below.

$$s^2\mathcal{L}(y_1) + s^2\mathcal{L}(y_2) = \frac{1}{s-2}$$
$$s\mathcal{L}(y_1) + s^2\mathcal{L}(y_2) = -\frac{1}{s-2}$$

$$\left[\begin{array}{cc|c} s^2 & s^2 & \frac{1}{s-2} \\ s & s^2 & -\frac{1}{s-2} \end{array}\right] \sim \left[\begin{array}{cc|c} s & s^2 & -\frac{1}{s-2} \\ s^2 & s^2 & \frac{1}{s-2} \end{array}\right] -sR1+R2$$

$$\sim \left[\begin{array}{cc|c} s & s^2 & -\frac{1}{s-2} \\ 0 & s^2-s^3 & \frac{1}{s-2} + \frac{s}{s-2} \end{array}\right] \quad (s^2-s^3)\mathcal{L}(y_2) = \frac{1}{s-2} + \frac{s}{s-2}$$

$$*_1 \quad \mathcal{L}(y_2) = \frac{1}{s^2(1-s)}\left[\frac{1}{s-2} + \frac{s}{s-2}\right] = \frac{-1}{s^2(s-1)(s-2)} - \frac{1}{s(s-1)(s-2)}$$

$$*_2 \quad s^2\mathcal{L}(y_1) + s^2\mathcal{L}(y_2) = s^2\mathcal{L}(y_1) + s^2\left[\frac{1}{s^2(1-s)}\left[\frac{1}{s-2} + \frac{s}{s-2}\right]\right] = \frac{1}{s-2}$$

This is getting very cumbersome! Not only *that*, but I *also* miscopied the problem. Oh, the y1 and y2 stuff is fine. Just re-labelling things with indexed variables rather than different letters designating different functions.

It's a lot easier to handle y1, y2, ..., y11 and keep things organized.

How about we try some Maple?

I would've done systems (Section 7.6) if there were some excitement over the linear algebra stuff. You *should* see these systems in Diff Eq with Linear Algebra.

$$x(t) = \frac{1}{2} + t\,e^{2t} - \frac{e^{2t}}{2}, y(t) = \frac{3}{4} - \frac{t}{2} - \left(e^{t}\right)^2 t + \frac{3\,e^{2t}}{4}.$$

Click Here for Maple for #9

Click Here for PDF of Maple for #9