#10

$with(inttrans)$

$[addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable, setup]$ **(1.1)**

$unassign('y(t)')$

$L := \frac{1}{2}; R := 10; C := \frac{1}{100};$

$$L := \frac{1}{2}$$

$$R := 10$$

$$C := \frac{1}{100} \qquad \textbf{(1.2)}$$

$eqn := L\cdot diff(x(t), t) + R\cdot x(t) + \frac{1}{C}\cdot\int_0^t x(\mathrm{tau})\ \mathrm{dtau} = 10 - 10\cdot \mathrm{Heaviside}(t-5)$

$$eqn := \frac{\frac{\mathrm{d}}{\mathrm{d}t} x(t)}{2} + 10\,x(t) + 100\left(\int_0^t x(\tau)\ \mathrm{d}\tau\right) = 10 - 10\ \mathrm{Heaviside}(t-5) \qquad \textbf{(1.3)}$$

$convert(\mathrm{Heaviside}(t-1), piecewise)$

$$\begin{cases} 0 & t < 1 \\ 1 & 1 \le t \end{cases} \qquad \textbf{(1.4)}$$

To make Heaviside function = 1 at 0:

$NumericEventHandler(invalid\_operation = `Heaviside/EventHandler`(value\_at\_zero = 1)) :$

$$t\,\mathrm{e}^{-2t} \qquad \textbf{(1.5)}$$

$laplace(eqn, t, s)$

$$\frac{s\,\mathcal{L}(x(t), t, s)}{2} - \frac{x(0)}{2} + 10\,\mathcal{L}(x(t), t, s) + \frac{100\,\mathcal{L}(x(t), t, s)}{s} = \frac{10\left(1 - \mathrm{e}^{-5s}\right)}{s} \qquad \textbf{(1.6)}$$

$solve(\%, laplace(x(t), t, s))$

$$-\frac{-x(0)\,s + 20\,\mathrm{e}^{-5s} - 20}{s^2 + 20\,s + 200} \qquad \textbf{(1.7)}$$

$subs(x(0\ ) = 0, \%)$

$$-\frac{-20 + 20\,\mathrm{e}^{-5s}}{s^2 + 20\,s + 200} \qquad \textbf{(1.8)}$$

$invlaplace(\%, s, t)$

$$2\,\mathrm{e}^{-10t}\sin(10\,t) - 2\ \mathrm{Heaviside}(t-5)\,\mathrm{e}^{-10t+50}\sin(10\,t-50) \qquad \textbf{(1.9)}$$

convert(%, *piecewise*)

$$\begin{cases} 2\,\mathrm{e}^{-10\,t}\sin(10\,t) & t<5 \\ 2\,\mathrm{e}^{-10\,t}\sin(10\,t)-2\,\mathrm{e}^{-10\,t+50}\sin(10\,t-50) & 5\le t \end{cases} \tag{1.10}$$

simplify(%)

$$2\,\mathrm{e}^{-10\,t}\sin(10\,t)-\left(\begin{cases} 0 & t<5 \\ 2\,\mathrm{e}^{-10\,t+50}\sin(10\,t-50) & 5\le t \end{cases}\right) \tag{1.11}$$

$$\int_0^t 2\,\mathrm{e}^{-10\,\mathrm{tau}}\sin(10\,\mathrm{tau})-\left(\begin{cases} 0 & \mathrm{tau}<5 \\ 2\,\mathrm{e}^{-10\,\mathrm{tau}+50}\sin(10\,\mathrm{tau}-50) & 5\le \mathrm{tau} \end{cases}\right)\mathrm{dtau}$$

$$\frac{1}{10}-\frac{\mathrm{e}^{-10\,t}\cos(10\,t)}{10}-\frac{\mathrm{e}^{-10\,t}\sin(10\,t)}{10}-\left(\begin{cases} 0 & t\le 5 \\ -\frac{\mathrm{e}^{-10\,t+50}\cos(10\,t-50)}{10}-\frac{\mathrm{e}^{-10\,t+50}\sin(10\,t-50)}{10}+\frac{1}{10} & 5<t \end{cases}\right) \tag{1.12}$$

convert(%, Heaviside)

$$\frac{1}{10}-\frac{\mathrm{e}^{-10\,t}\cos(10\,t)}{10}-\frac{\mathrm{e}^{-10\,t}\sin(10\,t)}{10}+\frac{\mathrm{Heaviside}(t-5)\,\mathrm{e}^{-10\,t+50}\cos(10\,t-50)}{10}+\frac{\mathrm{Heaviside}(t-5)\,\mathrm{e}^{-10\,t+50}\sin(10\,t-50)}{10}-\frac{\mathrm{Heaviside}(t-5)}{10} \tag{1.13}$$