# Section 7.4 Maple Stuff

Load packages:

$with(plots):$
$with(inttrans):$

## Example 2 - A command-line method for solving an ODE with Laplace Transforms

We solve $y'' + 16\,y = \cos(4\,t)$.

I thought I showed this to Erik on Thursday, 11/21, but all I got on ZOOM recording was the audio.

$eqn := diff(diff(y(t), t), t) + 16 \cdot y(t) = \cos(4 \cdot t)$

$$eqn := \frac{d^2}{dt^2}\, y(t) + 16\, y(t) = \cos(4\, t) \qquad \textbf{(1.1)}$$

$laplace(eqn, t, s)$

$$s^2\, \mathcal{L}(y(t), t, s) - D(y)(0) - s\, y(0) + 16\, \mathcal{L}(y(t), t, s) = \frac{s}{s^2 + 16} \qquad \textbf{(1.2)}$$

I copied the expression $L(y(t), t, s)$ and Maple wrote out the word "*laplace*," when I pasted it into the command line, below:

$solve(\%, laplace(y(t), t, s))$

$$\frac{y(0)\, s^3 + D(y)(0)\, s^2 + 16\, s\, y(0) + 16\, D(y)(0) + s}{\left(s^2 + 16\right)^2} \qquad \textbf{(1.3)}$$

$subs(y(0) = 0, \%)$

$$\frac{D(y)(0)\, s^2 + 16\, D(y)(0) + s}{\left(s^2 + 16\right)^2} \qquad \textbf{(1.4)}$$

In class, I didn't get the syntax right for $D(y)(0)$ in the following command. You can actually just copy it from the Maple output, above, and it's perfectly interpreted.

The initial condition on $y'$ is $y'(0) = 1$:

$subs(D(y)(0) = 1, \%)$

$$\frac{s^2 + s + 16}{\left(s^2 + 16\right)^2} \qquad \textbf{(1.5)}$$

$invlaplace(\%, s, t)$

$$\frac{\sin(4\,t)\,(2+t)}{8} \qquad \textbf{(1.6)}$$

# Example #4

Convolution of $e^t$ and $\sin(t)$.

$\int \exp(\mathrm{tau}) \cdot \sin(t - \mathrm{tau})\, \mathrm{dtau}$

$$\frac{e^{\tau}\cos(t-\tau)}{2} + \frac{e^{\tau}\sin(t-\tau)}{2} \qquad \textbf{(2.1)}$$

$\int_0^t \exp(\mathrm{tau}) \cdot \sin(t - \mathrm{tau})\, \mathrm{dtau}$

$$-\frac{\cos(t)}{2} - \frac{\sin(t)}{2} + \frac{e^{t}}{2} \qquad \textbf{(2.2)}$$

We find the Laplace transform of the expression:

$laplace(\%, t, s)$

$$\frac{1}{(s^2+1)\,(s-1)} \qquad \textbf{(2.3)}$$

$convert(\%, parfrac, s)$

$$\frac{1}{2\,(s-1)} + \frac{-s-1}{2\,(s^2+1)} \qquad \textbf{(2.4)}$$

# Random series command, because we were talking about long division of power series in class.

I didn't want to show you the "series" option in the "dsolve" command until we did things by hand, but this is a quick way to clobber it.

$series\left(\frac{\sin(x)}{\cos(x)}, x, 8\right)$

$$x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + O(x^9) \qquad \textbf{(3.1)}$$

$convert(\%, polynom)$

$$x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 \qquad \textbf{(3.2)}$$

$eqn := diff(diff(y(t), t), t) - 2 \cdot diff(y(t), t) + y(t) = 0$

$$eqn := \frac{d^2}{dt^2}\, y(t) - 2\, \frac{d}{dt}\, y(t) + y(t) = 0 \tag{3.3}$$

$dsolve(eqn, y(t), series)$

$$y(t) = y(0) + D(y)(0)\, t + \left(D(y)(0) - \frac{y(0)}{2}\right) t^2 + \left(\frac{D(y)(0)}{2} - \frac{y(0)}{3}\right) t^3 + \left(\frac{D(y)(0)}{6} - \frac{y(0)}{8}\right) t^4 + \left(\frac{D(y)(0)}{24} - \frac{y(0)}{30}\right) t^5 + O(t^6) \tag{3.4}$$

$subs(y(0) = 1, \%)$

$$y(t) = 1 + D(y)(0)\, t + \left(D(y)(0) - \frac{1}{2}\right) t^2 + \left(\frac{D(y)(0)}{2} - \frac{1}{3}\right) t^3 + \left(\frac{D(y)(0)}{6} - \frac{1}{8}\right) t^4 + \left(\frac{D(y)(0)}{24} - \frac{1}{30}\right) t^5 + O(t^6) \tag{3.5}$$

$subs(D(y)(0) = 1, \%)$

$$y(t) = 1 + t + \frac{1}{2}\, t^2 + \frac{1}{6}\, t^3 + \frac{1}{24}\, t^4 + \frac{1}{120}\, t^5 + O(t^6) \tag{3.6}$$

# Example 6

$$\frac{1}{k^2} \cdot \int_0^t \sin(k \cdot \mathrm{tau}) \cdot \sin(t - \mathrm{tau})\, d\mathrm{tau}$$

$$-\frac{-\sin(t)\, k + \sin(k\, t)}{k^2\, (k-1)\, (k+1)} \tag{4.1}$$

# Example 8

We solve the integro-differential equation $L\frac{di}{dt} + Ri + \frac{1}{C}\int_0^t i(\mathrm{tau})\, d\mathrm{tau} = E(t)$. The voltage drop of the circuit is the sum of the voltage drops contributed by inductance *L*, resistance *R*, and capacitance *C*.

Here, $E(t) = 120\, t - 120\, t \cdot \mathrm{Heaviside}(t - 1)$.

Load *inttrans* package, for we wish to use *laplace* and *invlaplace* commands:

$with(inttrans)$

[*addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable, setup*] **(5.1)**

$unassign('y(t)')$

To make Heaviside function = 1 at 0:

$NumericEventHandler(invalid\_operation = `Heaviside/EventHandler`(value\_at\_zero = 0)) :$

To un-do this:

$NumericEventHandler(invalid\_operation = default) :$

Heaviside(*0*)

$$1 \tag{5.2}$$

Alternative to the EventHandler thing:

$\_EnvUseHeavisideAsUnitStep := true$

$$\_EnvUseHeavisideAsUnitStep := true \tag{5.3}$$

Heaviside(0)

$$1 \tag{5.4}$$

$E := t \to 120 \cdot t - 120 \cdot t \cdot \mathrm{Heaviside}(t-1)$

$$E := t \mapsto 120 \cdot t - 120 \cdot t \cdot \mathrm{Heaviside}(t-1) \tag{5.5}$$

$eqn := L \cdot diff(y(t), t) + R \cdot y(t) + \frac{1}{C} \cdot \int_0^t y(\mathrm{tau})\, \mathrm{dtau} = E(t)$

$$eqn := L \left( \frac{\mathrm{d}}{\mathrm{d}t} y(t) \right) + R\, y(t) + \frac{\int_0^t y(\tau)\, \mathrm{d}\tau}{C} = 120\, t - 120\, t\, \mathrm{Heaviside}(t-1) \tag{5.6}$$

$subs\left(L = \frac{1}{10}, R = 2, C = \frac{1}{10}, eqn\right)$

$$\frac{\frac{\mathrm{d}}{\mathrm{d}t} y(t)}{10} + 2\, y(t) + 10 \left( \int_0^t y(\tau)\, \mathrm{d}\tau \right) = 120\, t - 120\, t\, \mathrm{Heaviside}(t-1) \tag{5.7}$$

$laplace(\%, t, s)$

$$\frac{s\, \mathcal{L}(y(t), t, s)}{10} - \frac{y(0)}{10} + 2\, \mathcal{L}(y(t), t, s) + \frac{10\, \mathcal{L}(y(t), t, s)}{s} = \frac{120\, (1 - \mathrm{e}^{-s}\, (s+1))}{s^2} \tag{5.8}$$

$subs(y(0\ ) = 0, \%)$

$$\frac{s\, \mathcal{L}(y(t), t, s)}{10} + 2\, \mathcal{L}(y(t), t, s) + \frac{10\, \mathcal{L}(y(t), t, s)}{s} = \frac{120\, (1 - \mathrm{e}^{-s}\, (s+1))}{s^2} \tag{5.9}$$

$solve(\%, laplace(y(t), t, s))$

$$-\frac{1200\, (\mathrm{e}^{-s}\, s + \mathrm{e}^{-s} - 1)}{s\, (s^2 + 20\, s + 100)} \tag{5.10}$$

$invlaplace(\%, s, t)$

$$12\, \mathrm{Heaviside}(1-t) - 12\, \mathrm{Heaviside}(t-1)\, \mathrm{e}^{-10\, t + 10}\, (90\, t - 91) - 12\, \mathrm{e}^{-10\, t}\, (10\, t + 1) \tag{5.11}$$

$convert(\%, piecewise)$

$$\begin{cases} -12\,e^{-10\,t}\,(10\,t+1)+12 & t<1 \\ -132\,e^{-10}+24 & t=1 \\ -12\,e^{-10\,t}\,(10\,t+1)-12\,e^{-10\,t+10}\,(90\,t-91) & 1<t \end{cases} \qquad \textbf{(5.12)}$$

*convert*(%, *piecewise*)

$$\begin{cases} -12\,e^{-10\,t}\,(10\,t+1)+12 & t<1 \\ -132\,e^{-10}+24 & t=1 \\ -12\,e^{-10\,t}\,(10\,t+1)-12\,e^{-10\,t+10}\,(90\,t-91) & 1<t \end{cases} \qquad \textbf{(5.13)}$$

OK. This is Maple's version. It's not in 1-to-1 correspondence with the book, and I've run out of patience with the example.