CHEAT SHEET Ch 7

$\boxed{\text{D 7.1.1}}$ f defined on $[0,\infty) \Longrightarrow F(s) = \int_0^\infty e^{-st} f(t)\,dt$ is the Laplace Transform

$\mathcal{D}(F) = \{s \mid \text{the integral converges}\}$.

f must be piecewise-cnts

$\mathcal{L}(1) = \frac{1}{s}$, $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$, $\mathcal{L}(e^{at}) = \frac{1}{s-a}$, $\mathcal{L}(\sin(kt)) = \frac{1}{s^2+k^2}$

$\mathcal{L}(\cos(kt)) = \frac{s}{s^2+k^2}$, $\mathcal{L}(\sinh(kt)) = \frac{k}{s^2-k^2}$, $\mathcal{L}(\cosh(kt)) = \frac{s}{s^2-k^2}$

$\boxed{\text{D 7.1.2}}$ f defined on $[0,\infty)$ is of EXPONENTIAL ORDER if $\exists\, c, M, T > 0 \ni$

$|f(t)| \le Me^{ct} \;\forall\, t > T$.

$\boxed{\text{T7.1.2}}$ f pcws-cnts on $[0,\infty)$ of exponential order $\Longrightarrow \mathcal{L}(f(t)) \,\exists\, \forall\, s > c$.

$\boxed{\text{T7.1.3}}$ $F(s) = \mathcal{L}(f(t))$, where f is of exponential order $\Longrightarrow F(s) \xrightarrow{s\to\infty} 0$.

$\mathcal{L}^{-1}\left(\frac{1}{s}\right) = 1$, $\mathcal{L}^{-1}\left(\frac{n!}{s^{n+1}}\right) = t^n$, $\mathcal{L}^{-1}\left(\frac{1}{s-a}\right) = e^{at}$, $\mathcal{L}^{-1}\left(\frac{k}{s^2+k^2}\right) = \sin(kt)$, $\mathcal{L}^{-1}\left(\frac{s}{s^2+k^2}\right) = \cos(kt)$

$\mathcal{L}^{-1}\left(\frac{1}{s^n}\right) = \frac{t^{n-1}}{(n-1)!}$, $\mathcal{L}^{-1}\left(\frac{1}{s^2+k^2}\right) = \frac{1}{k}\sin(kt)$

$\boxed{\text{T7.2.2}}$ $f, f^{(1)}, \dots, f^{(n-1)}$ cnts on $[0,\infty)$, $f^{(n)}$ pcws-cnts on $[0,\infty)$, $f^{(k)}$ is E.O., $k=0,\dots,n$, and c as in $\boxed{\text{D7.1.2}} \Longrightarrow \mathcal{L}(f^{(n)}(t)) = s^n \mathcal{L}(f(t)) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$

$\boxed{\text{T7.3.1}}$ $\mathcal{L}(f(t)) = F(s)\ \forall\, s > 0$ & $a \in \mathbb{R} \Longrightarrow \mathcal{L}(e^{at}f(t)) = F(s-a)$

$\boxed{\text{D7.3.1}}$ UNIT STEP FUNCTION (Heaviside Function)

$$\mathcal{U}(t-a) = \begin{cases} 0 & \text{if } 0 \le t < a \\ 1 & \text{if } a \le t \end{cases}$$

$\boxed{\text{T7.3.2}}$ $\mathcal{L}(f(t)) = F(s)\ \exists$ for $s > c \ge 0$ & $a > 0 \Longrightarrow \mathcal{L}(f(t-a)\mathcal{U}(t-a)) = e^{-as}F(s)$

$\boxed{\text{T7.3.2}}$ Inverse Form: $\mathcal{L}^{-1}(e^{-as}F(s)) = f(t-a)\mathcal{U}(t-a)$

Also: $\mathcal{L}(g(t)\mathcal{U}(t-a)) = e^{-as}\mathcal{L}(g(t+a))$

$\boxed{\text{T7.4.1}}$ f pcws-cnts & E.O. on $[0,\infty)$ & $\mathcal{L}(f(t)) = F(s) \Longrightarrow$

$\mathcal{L}(t^n f(t)) = (-1)^n \frac{d^n}{ds^n}[F(s)]$.

$f * g = \int_0^t f(\tau)g(t-\tau)\,d\tau$ CONVOLUTION

$\boxed{\text{T7.4.2}}$ $\mathcal{L}(f*g) = F(s)G(s)$

INVERSE FORM: $\mathcal{L}^{-1}(F(s)G(s)) = f*g$