Section 6.1 #6

Consider the following differential equation to be solved using a power series as in Example 4.

6. $y' = xy$

Using the substitution $y = \sum_{n=0}^{\infty} c_n x^n$, find an expression for the following coefficients. (Give your answers in terms of c_0.)

$c_2 = (\quad)c_0$

$c_3 = (\quad)c_0$

$c_4 = (\quad)c_0$

$c_5 = (\quad)c_0$

$c_6 = (\quad)c_0$

$$y' = \sum_{n=0}^{\infty} c_n \cdot n x^{n-1} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4$$

$$= c_0 \cdot 0 \cdot x^{-1} + c_1 + 2c_2 x^1 + 3c_3 x^2 + 4c_4 x^3 + \ldots$$

$$= c_1 x^0 + 2c_2 x^1 + 3c_3 x^2 + \ldots$$

$n=0 \quad n=1 \quad n=2$

$$= c_{n+1} x^n + 2c_{n+1} x^n + (n+1) c_{n+1} x^n$$

$$y' = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n$$

Find the solution. (Give your answer in terms of c_0.)

$$y(x) = c_0 \sum_{n=0}^{\infty} \left(\quad \right) \qquad \frac{2^{-n}(x^2)^n}{n!}$$

$$xy = x \sum_{n=0}^{\infty} c_n x^n = \sum_{n=0}^{\infty} c_n x^{n+1}$$

$$xy = c_0 x^1 + c_1 x^2 + c_2 x^3 + c_3 x^4 + c_4 x^5$$

$$= y' = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + 6c_6 x^5$$

$\Longrightarrow c_1 = 0 = c_3 = c_5 = \cdots$

x: $c_0 = 2c_2 \Longrightarrow c_2 = \frac{1}{2}c_0$

x^3: $c_2 = 4c_4 \longrightarrow c_4 = \frac{1}{4}\underset{n=2}{c_2} = \frac{1}{4}\cdot\frac{1}{2}c_0\, x^3 = x^{2n-1}$

x^5: $c_4 = 6c_6 \Longrightarrow c_6 = \frac{1}{6}\cdot c_4 = \frac{1}{6}\cdot\frac{1}{4}\cdot\frac{1}{2}\cdot c_0$

$= \frac{1}{2^3}\cdot\frac{1}{3!}c_0\, x^5$

$n=3$

$5 = 2(3) - 1$

~~$\sum \frac{1}{2^n}\cdot\frac{1}{n!}c_0\, x^{2n-1}$~~

Go back to

$y = \sum_{n=0}^{\infty} c_n x^n$

$c_{2k} =$

$= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \cdots$

$= c_0 + \frac{1}{2}c_0 x^2 + \frac{1}{2^2}\cdot\frac{1}{2!}x^4 + \frac{1}{2^3}\frac{1}{3!}x^6 + \cdots$

$$= c_0 \sum_{n=0}^{\infty} \frac{1}{2^n}\cdot\frac{1}{n!}x^{2n}$$