(6) $y'' + y = \sec^2(x)$

$r^2 + 1 = 0 \Rightarrow r = \pm i \Rightarrow y_h = c_1\cos(x) + c_2\sin(x)$

$y_1 = \cos(x),\ y_2 = \sin(x)$

$|A| = W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_1'y_2$

$= \cos(x)(\cos(x) - (-\sin(x))(\sin(x))) = \boxed{1 = W(y_1, y_2)}$

$G(x,t) = \dfrac{y_1(t)y_2(x) - y_1(x)y_2(t)}{y_1(t)y_2'(t) - y_1'(t)y_2(t)}$

$= \dfrac{\cos(t)\sin(x) - \cos(t)\sin(t)}{1}$

$\Rightarrow y_p = \int_{x_0}^{x} G(t,t)\, f(t)\, dt = \int_{x_0}^{x} \left(\cos(t)\sin(x) - \cos(x)\sin(t)\right)\sec^2(t)\, dt$

$= \int_{x_0}^{x} \left(\sin(x)\sec(t) - \cos(x)(\sec(t)\tan(t))\right) dt$

$= \ln|\sec(t) + \tan(t)|\Big|_{x_0}^{x} \sin(x) - \cos(x)\left[\sec(t)\Big|_{x_0}^{x}\right]$

$= \sin(x)\ln|\sec(x) + \tan(x)| - \cos(x)(\sec(x) - \sec(x_0))$

$= \sin(x)\ln|\sec(x) + \tan(x)| - 1 + \dfrac{\cos(x)}{\sec(\pi)} =$

$= \sin(x)\ln|\sec(x) + \tan(x)| - \cos(x) - 1 = y_p$

Now for the constants in y_h:

$y_h = c_1\cos(x) + c_2\sin(x)$

$y_h(\pi) = \frac{1}{5} = c_1\cos\pi + c_2\sin\pi = c_1(-1) \Rightarrow$

$\boxed{c_1 = -\frac{1}{5}}$

$y_h' = -c_1\sin(x) + c_2\cos(x)$

$y_h'(\pi) = -1 = c_2\cos(\pi) = -c_2 \Rightarrow c_2 = 1$

$y_h = -\frac{1}{5}\cos(x) + \sin(x)$

$y_h + y_p = -\frac{1}{5}\cos(x) + \sin(x) + \sin(x)\ln|\sec(x) + \tan(x)| - 1 + \dfrac{\cos(x)}{\sec(\pi)} =$