$with(Student[Calculus1])$

$[AntiderivativePlot, AntiderivativeTutor, ApproximateInt, ApproximateIntTutor, ArcLength, ArcLengthTutor, Asymptotes, Clear, CriticalPoints, CurveAnalysisTutor, DerivativePlot, DerivativeTutor, DiffTutor, Distance, ExtremePoints, FunctionAverage, FunctionAverageTutor, FunctionChart, FunctionPlot, GetMessage, GetNumProblems, GetProblem, Hint, InflectionPoints, IntTutor, Integrand, InversePlot, InverseTutor, LimitTutor, MeanValueTheorem, MeanValueTheoremTutor, NewtonQuotient, NewtonsMethod, NewtonsMethodTutor, PointInterpolation, RiemannSum, RollesTheorem, Roots, Rule, Show, ShowIncomplete, ShowSolution, ShowSteps, Summand, SurfaceOfRevolution, SurfaceOfRevolutionTutor, Tangent, TangentSecantTutor, TangentTutor, TaylorApproximation, TaylorApproximationTutor, Understand, Undo, VolumeOfRevolution, VolumeOfRevolutionTutor, WhatProblem]$ **(1)**

?*TaylorApproximation*

This is my first pass at Power Series in Diff Eq. TaylorApproximation looks like it might give us what we want. It's all primed to display a plot of *f* and its Taylor Series approximation of whatever degree or span of degrees you choose. I think we'll be more interested in the Taylor Polynomials, themselves. But just getting started on Chapter 6.

We multiply the first 20 terms of e^x and $\sin(x)$. Crude but effective

Sometimes the partials sums won't be displayed in either ascending or descending order, unless you ask in a nice way. The **sort** command will do this for you. Syntax for the command given below. You can hit "enter" on the *?sort* line that I inserted and see the man pages for **sort**.

I built these polynomials using the Expressions palette on the left panel. You can enter sums in a very intuitive way, if you know how to use sigma notation.

Here, I'm adding partial sums for e^x and $\sin(x)$.

$$f := x \to \sum_{n=0}^{7}\left(\frac{x^n}{(n)!}\right)$$

$$f := x \mapsto \sum_{n=0}^{7}\frac{x^n}{n!} \quad \textbf{(1.1)}$$

$f(x)$

$$1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 \quad \textbf{(1.2)}$$

$$g := x \to \sum_{n=0}^{7}\left(\frac{(-1)^n x^{2\cdot n+1}}{(2\cdot n+1)!}\right)$$

$$g := x \mapsto \sum_{n=0}^{7} \frac{(-1)^n \cdot x^{2 \cdot n + 1}}{(2 \cdot n + 1)!} \tag{1.3}$$

$g(x)$

$$x - \frac{1}{6} x^3 + \frac{1}{120} x^5 - \frac{1}{5040} x^7 + \frac{1}{362880} x^9 - \frac{1}{39916800} x^{11} + \frac{1}{6227020800} x^{13} - \frac{1}{1307674368000} x^{15} \tag{1.4}$$

$h := x \to f(x) + g(x)$

$$h := x \mapsto f(x) + g(x) \tag{1.5}$$

?*sort*

$h(x)$

$$1 + 2x + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{60} x^5 + \frac{1}{720} x^6 + \frac{1}{362880} x^9 - \frac{1}{39916800} x^{11} + \frac{1}{6227020800} x^{13} - \frac{1}{1307674368000} x^{15} \tag{1.6}$$

$sort(h(x), x, ascending)$

$$1 + 2x + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{60} x^5 + \frac{1}{720} x^6 + \frac{1}{362880} x^9 - \frac{1}{39916800} x^{11} + \frac{1}{6227020800} x^{13} - \frac{1}{1307674368000} x^{15} \tag{1.7}$$

6.1 #4

You'll definitely enjoy the **TaylorPolynomial** command. But you should be pretty good just entering sums using the palette, like I did, above.

?*TaylorPolynomial*
with(*Student*[*Calculus1*]) :
$h := x \to TaylorApproximation(\sin(x), x = 0, degree = 7)$

$$h := x \mapsto TaylorApproximation(\sin(x), x = 0, degree = 7) \tag{1.1.1}$$

$h(x)$

$$x - \frac{1}{6} x^3 + \frac{1}{120} x^5 - \frac{1}{5040} x^7 \tag{1.1.2}$$

$k := x \to TaylorApproximation(\cos(x), x = 0, degree = 7)$

$$k := x \mapsto TaylorApproximation(\cos(x), x = 0, degree = 7) \tag{1.1.3}$$

$k(x)$

$$1-\frac{1}{2}x^2+\frac{1}{24}x^4-\frac{1}{720}x^6 \quad \textbf{(1.1.4)}$$

$r := x \rightarrow h(x) \cdot k(x)$

$$r := x \mapsto h(x) \cdot k(x) \quad \textbf{(1.1.5)}$$

$r(x)$

$$\left(x-\frac{1}{6}x^3+\frac{1}{120}x^5-\frac{1}{5040}x^7\right)\left(1-\frac{1}{2}x^2+\frac{1}{24}x^4-\frac{1}{720}x^6\right) \quad \textbf{(1.1.6)}$$

Notice there are only odd powers, and the power of the 4th term in the product is of degree 7! That means you'd have to be sure to capture the products of the
x^7 and x^0 terms, the x^5 and x^2 terms, the x^3 and x^4 terms, and the x and x^6 terms.

$sort(expand(r(x)), x, ascending)$

$$x-\frac{2}{3}x^3+\frac{2}{15}x^5-\frac{4}{315}x^7+\frac{41}{60480}x^9-\frac{1}{50400}x^{11}+\frac{1}{3628800}x^{13} \quad \textbf{(1.1.7)}$$

Just clobbering the product, directly, with TaylorApproximation seems like cheating.

I'm not sure if I'll ask you to turn the crank on one of these products on a test or not.

I wonder if the class wants or needs more work on partial fractions...

$TaylorApproximation(\sin(x) \cdot \cos(x), degree = 11)$

$$x-\frac{2}{3}x^3+\frac{2}{15}x^5-\frac{4}{315}x^7+\frac{2}{2835}x^9-\frac{4}{155925}x^{11} \quad \textbf{(1.1.8)}$$

Grind it out by the definition - Didn't Go Far Enough!

Here we are reminded that carrying out the multiplication on the first 4 terms of each of the factors does *not* capture the first 4 terms of the product. Here, the first 4 terms of the product contain

$f := x \rightarrow \sin(x)$

$$f := x \mapsto \sin(x) \quad \textbf{(1.1.1.1)}$$

$fp := \mathrm{D}(f)$

$$fp := \cos \quad \textbf{(1.1.1.2)}$$

$fpp := \mathrm{D}(fp)$

$$fpp := -\sin \quad \textbf{(1.1.1.3)}$$

$fppp := \mathrm{D}(fpp)$

$$fppp := -\cos \quad \textbf{(1.1.1.4)}$$

$fpppp := \mathrm{D}(fppp)$

$$fpppp := \sin \quad \textbf{(1.1.1.5)}$$

$g := x \rightarrow \cos(x)$

$$g := x \mapsto \cos(x) \quad \textbf{(1.1.1.6)}$$

$gp := \mathrm{D}(g)$

$$gp := x \mapsto -\sin(x) \quad \textbf{(1.1.1.7)}$$

$gpp := D(gp)$

$$gpp := x \mapsto -\cos(x) \quad \textbf{(1.1.1.8)}$$

$gppp := D(gpp)$

$$gppp := \sin \quad \textbf{(1.1.1.9)}$$

$gpppp := D(gppp)$

$$gpppp := \cos \quad \textbf{(1.1.1.10)}$$

$mysinepolynomial := x \to f(0) + fp(0) \cdot x + \frac{fpp(0) \cdot x^2}{2} + \frac{fppp(0) \cdot x^3}{(3)!} + \frac{fpppp(0) \cdot x^4}{(4)!}$

$$mysinepolynomial := x \mapsto f(0) + fp(0) \cdot x + \frac{fpp(0) \cdot x^2}{2} + \frac{fppp(0) \cdot x^3}{3!} + \frac{fpppp(0) \cdot x^4}{4!} \quad \textbf{(1.1.1.11)}$$

$mycosinepolynomial := x \to g(0) + gp(0) \cdot x + \frac{gpp(0) \cdot x^2}{2} + \frac{gppp(0) \cdot x^3}{(3)!} + \frac{gpppp(0) \cdot x^4}{(4)!}$

$$mycosinepolynomial := x \mapsto g(0) + gp(0) \cdot x + \frac{gpp(0) \cdot x^2}{2} + \frac{gppp(0) \cdot x^3}{3!} + \frac{gpppp(0) \cdot x^4}{4!} \quad \textbf{(1.1.1.12)}$$

$mycosinepolynomial(x) + mysinepolynomial(x)$

$$1 - \frac{1}{2} x^2 + \frac{1}{24} x^4 + x - \frac{1}{6} x^3 \quad \textbf{(1.1.1.13)}$$

The thing I'm doing wrong is I'm not using enough terms to capture the first 4 terms of the product, completely.

TaylorApproximation Command

$TaylorApproximation(\sin(x), degree = 11)$

$$x - \frac{1}{6} x^3 + \frac{1}{120} x^5 - \frac{1}{5040} x^7 + \frac{1}{362880} x^9 - \frac{1}{39916800} x^{11} \quad \textbf{(2.1)}$$

$TaylorApproximation(\sin(x), degree = 10)$

$$x - \frac{1}{6} x^3 + \frac{1}{120} x^5 - \frac{1}{5040} x^7 + \frac{1}{362880} x^9 \quad \textbf{(2.2)}$$

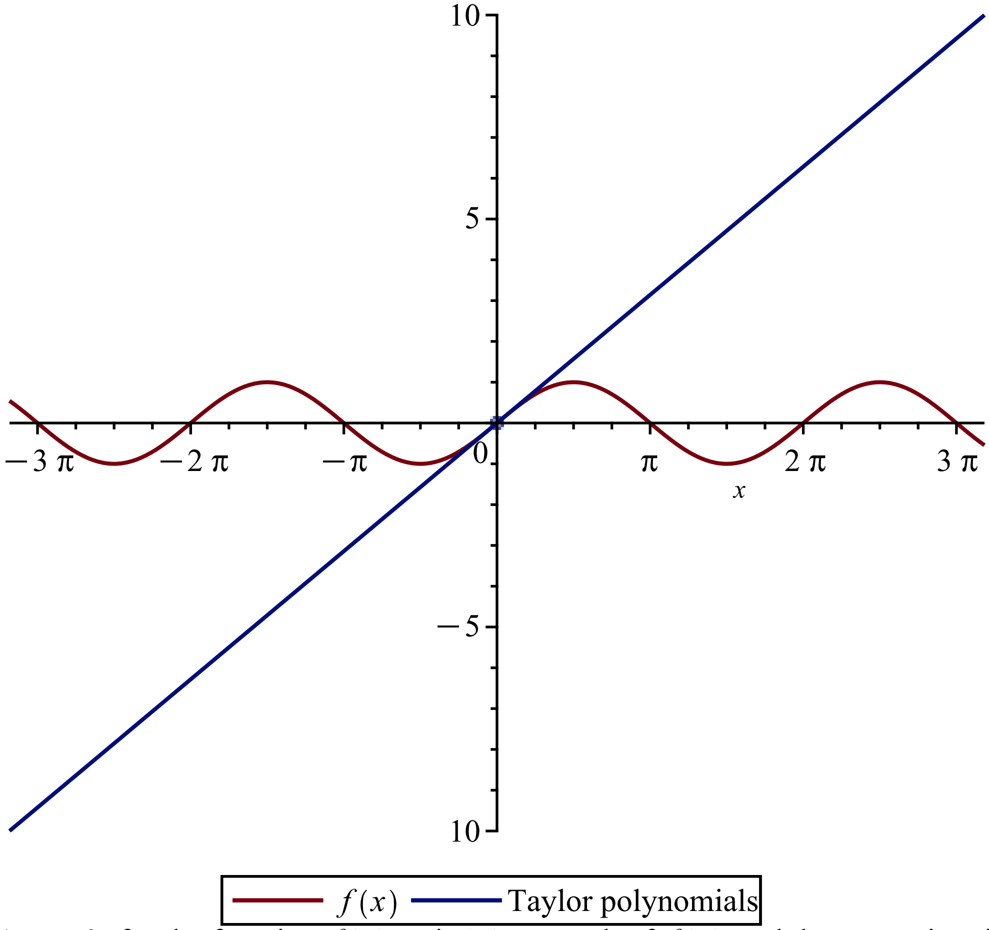
$TaylorApproximation(\sin(x), degree = 1..7)$

$$x, x, x - \frac{1}{6} x^3, x - \frac{1}{6} x^3, x - \frac{1}{6} x^3 + \frac{1}{120} x^5, x - \frac{1}{6} x^3 + \frac{1}{120} x^5, x - \frac{1}{6} x^3 + \frac{1}{120} x^5 \quad \textbf{(2.3)}$$

$$- \frac{1}{5040} x^7$$

$TaylorApproximation(\sin(x), degree = 1..20, view = [-10..10, -10..10], output = animation)$

Taylor Polynomials - Animation



At $x = 0$, for the function $f(x) = \sin(x)$, a graph of $f(x)$ and the approximati

$f := x \to TaylorApproximation(\sin(x), degree = 15)$

$$f := x \mapsto TaylorApproximation(\sin(x), degree = 15) \quad \textbf{(2.4)}$$

$f(x)$

$$x - \frac{1}{6} x^3 + \frac{1}{120} x^5 - \frac{1}{5040} x^7 + \frac{1}{362880} x^9 - \frac{1}{39916800} x^{11} + \frac{1}{6227020800} x^{13} - \frac{1}{1307674368000} x^{15} \quad \textbf{(2.5)}$$

$g := x \to TaylorApproximation(\cos(x), degree = 15)$

$$g := x \mapsto TaylorApproximation(\cos(x), degree = 15) \quad \textbf{(2.6)}$$

$g(x)$

$$1 - \frac{1}{2} x^2 + \frac{1}{24} x^4 - \frac{1}{720} x^6 + \frac{1}{40320} x^8 - \frac{1}{3628800} x^{10} + \frac{1}{479001600} x^{12} - \frac{1}{87178291200} x^{14} \quad \textbf{(2.7)}$$