$with(VectorCalculus):$
$with(LinearAlgebra):$

#6 - Implementing the Particular Solution, given the solutions to the homogeneous equation.

This little sub-module is designed to crank out solutions for multiple right-hand sides. This is one way to achieve what Green's Functions will achieve in the sequel in
Section 4.8.

$eqn1 := 3\cdot x^2\cdot diff(diff(y(x), x), x) + 7\cdot x\cdot diff(y(x), x) + y(x) = 0$

$$eqn1 := 3\,x^2\left(\frac{d^2}{dx^2}\,y(x)\right) + 7\,x\left(\frac{d}{dx}\,y(x)\right) + y(x) = 0 \tag{1.1}$$

$eqn2 := 3\cdot x^2\cdot diff(diff(y(x), x), x) + 7\cdot x\cdot diff(y(x), x) + y(x) = x^2 - x$

$$eqn2 := 3\,x^2\left(\frac{d^2}{dx^2}\,y(x)\right) + 7\,x\left(\frac{d}{dx}\,y(x)\right) + y(x) = x^2 - x \tag{1.2}$$

$dsolve(eqn1, y(x))$

$$y(x) = \frac{\_C1}{x} + \frac{\_C2}{x^{1/3}} \tag{1.3}$$

$dsolve(eqn2, y(x))$

$$y(x) = \frac{\_C1}{x} + \frac{\_C2}{x^{1/3}} + \frac{x\,(8\,x - 21)}{168} \tag{1.4}$$

$expand(\%)$

$$y(x) = \frac{\_C1}{x} + \frac{\_C2}{x^{1/3}} + \frac{x^2}{21} - \frac{x}{8} \tag{1.5}$$

$Wronskian\left(\left[\frac{1}{x^{\frac{1}{3}}}, \frac{1}{x}\right], x, 'determinant'\right)$

$$\begin{bmatrix} \frac{1}{x^{1/3}} & \frac{1}{x} \\ -\frac{1}{3\,x^{4/3}} & -\frac{1}{x^2} \end{bmatrix}, -\frac{2}{3\,x^{7/3}} \tag{1.6}$$

$W := x \rightarrow -\frac{2}{3\,x^{7/3}}$

$$W := x \mapsto -2\cdot\left(3\cdot x^{7\cdot 3^{-1}}\right)^{-1} \tag{1.7}$$

$W(x)$

$$-\frac{2}{3\,x^{7/3}} \tag{1.8}$$

$$A := \begin{bmatrix} \frac{1}{x^{1/3}} & \frac{1}{x} \\ -\frac{1}{3\,x^{4/3}} & -\frac{1}{x^2} \end{bmatrix}$$

$$A := \begin{bmatrix} \frac{1}{x^{1/3}} & \frac{1}{x} \\ -\frac{1}{3\,x^{4/3}} & -\frac{1}{x^2} \end{bmatrix} \tag{1.9}$$

$$neweqn2 := \frac{eqn2}{3 \cdot x^2}$$

$$neweqn2 := \frac{3\,x^2 \left(\frac{d^2}{dx^2}\,y(x)\right) + 7\,x\left(\frac{d}{dx}\,y(x)\right) + y(x)}{3\,x^2} = \frac{x^2 - x}{3\,x^2} \tag{1.10}$$

$$rhs(neweqn2)$$

$$\frac{x^2 - x}{3\,x^2} \tag{1.11}$$

$$f := x \rightarrow rhs(neweqn2)$$

$$f := x \mapsto rhs(neweqn2) \tag{1.12}$$

$$f(x)$$

$$\frac{x^2 - x}{3\,x^2} \tag{1.13}$$

$$A1 := \langle \langle 0, f(x) \rangle | Column(A, 2) \rangle$$

$$A1 := \begin{bmatrix} 0 & \frac{1}{x} \\ \frac{x^2 - x}{3\,x^2} & -\frac{1}{x^2} \end{bmatrix} \tag{1.14}$$

$$W1 := x \rightarrow Determinant(A1)$$

$$W1 := x \mapsto Determinant(A1) \tag{1.15}$$

$$W1(x)$$

$$-\frac{x - 1}{3\,x^2} \tag{1.16}$$

$$A2 := \langle Column(A, 1) | \langle 0, f(x) \rangle \rangle$$

$$A2 := \begin{bmatrix} \frac{1}{x^{1/3}} & 0 \\ -\frac{1}{3\,x^{4/3}} & \frac{x^2 - x}{3\,x^2} \end{bmatrix} \tag{1.17}$$

$W2 := x \to Determinant(A2)$

$$W2 := x \mapsto Determinant(A2) \tag{1.18}$$

$W2(x)$

$$\frac{x - 1}{3\,x^{4/3}} \tag{1.19}$$

$y1 := x \to x^{-\frac{1}{3}}$

$$y1 := x \mapsto x^{-3^{-1}} \tag{1.20}$$

$y2 := x \to x^{-1}$

$$y2 := x \mapsto \frac{1}{x} \tag{1.21}$$

$yp1 := x \to y1(x) \cdot \int \frac{W1(x)}{W(x)}\,\mathrm{d}x$

$$yp1 := x \mapsto y1(x) \cdot \left(\int W1(x) \cdot W(x)^{-1}\,\mathrm{d}x \right) \tag{1.22}$$

$simplify(y1(x))$

$$\frac{1}{x^{1/3}} \tag{1.23}$$

$yp2 := x \to y2(x) \cdot \int \frac{W2(x)}{W(x)}\,\mathrm{d}x$

$$yp2 := x \mapsto y2(x) \cdot \left(\int W2(x) \cdot W(x)^{-1}\,\mathrm{d}x \right) \tag{1.24}$$

$yp2(x)$

$$\frac{-\frac{1}{6}\,x^3 + \frac{1}{4}\,x^2}{x} \tag{1.25}$$

$simplify(yp2(x))$

$$-\frac{1}{6}\,x^2 + \frac{1}{4}\,x \tag{1.26}$$

$yp1(x) + yp2(x)$

$$\frac{3\,x\,(4\,x - 7)}{56} + \frac{-\frac{1}{6}\,x^3 + \frac{1}{4}\,x^2}{x} \tag{1.27}$$

$simplify(\%)$

$$\frac{1}{21} x^2 - \frac{1}{8} x \tag{1.28}$$