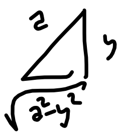
$$\frac{dy}{dx} = -\frac{y}{\sqrt{a^2-y^2}}$$

$$\int \frac{\sqrt{a^2-y^2}\,dy}{y} = \int -dx \longrightarrow \int \frac{\sqrt{a^2-a^2\sin^2\theta}}{a\sin\theta}\cdot a\cos\theta\,d\theta = -x + C$$



$y = a\sin\theta$

$dy = a\cos\theta\,d\theta$

$$= \int \frac{a\sqrt{1-\sin^2\theta}}{a\sin\theta}\cdot a\cos\theta\,d\theta$$

$$= a\int \frac{|\cos\theta|\cos\theta\,d\theta}{\sin\theta} = a\int \frac{\cos^2\theta}{\sin\theta}\,d\theta = a\int \frac{1-\sin^2\theta}{\sin\theta}\,d\theta = a\int(\csc\theta - \sin\theta)\,d\theta$$

$= I_1 + I_2$, where $I_2 = -a\int \sin\theta\,d\theta$

$$I_1 = a\int \csc\theta\,d\theta = \int\left(\frac{-\csc\theta-\cot\theta}{-\csc\theta-\cot\theta}\right)\csc\theta\,d\theta = -a\int \frac{-\csc^2\theta - \csc\theta\cot\theta}{\csc\theta+\cot\theta}\,d\theta$$

$$= -a\int\frac{du}{u} = -a\ln|u| + C = -a\ln|\csc\theta + \cot\theta| + C$$

you can play with this, to agree with Maple & WebAssign:

OPTIONAL

$$\csc\theta + \cot\theta = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \left(\frac{\cos\theta + 1}{\sin\theta}\right)\left(\frac{\cos\theta - 1}{\cos\theta - 1}\right)$$

$$= \frac{\cos^2\theta - 1}{\sin\theta(\cos\theta - 1)} = \frac{-\sin^2\theta}{\sin\theta(\cos\theta - 1)} = \frac{-\sin\theta}{\cos\theta - 1}$$

$$\Rightarrow -a\ln|\csc\theta + \cot\theta| = -a\ln\left(\frac{-\sin\theta}{\cos\theta - 1}\right) = a\ln\left(\frac{\cos\theta - 1}{-\sin\theta}\right)$$

$$a\ln\left(\frac{1-\cos\theta}{\sin\theta}\right) = a\ln(\csc\theta - \cot\theta)$$

$$= a\ln\left(\frac{a}{y} - \frac{\sqrt{a^2-y^2}}{y}\right)$$

$$= \text{BOOK} = \text{MAPLE}$$

$I_2 = a\int -\sin(x)\,dx =$

$a\cos(x) + C$

$I_1 + I_2 = -a\ln|\csc\theta + \cot\theta| + a\cos\theta + C$

$= -a\ln\left|\frac{a}{y} + \frac{\sqrt{a^2-y^2}}{y}\right| + a\frac{\sqrt{a^2-y^2}}{a} + C \quad = -a\ln\left|\frac{a}{y} + \frac{\sqrt{a^2-y^2}}{a}\right| + \sqrt{a^2-y^2} + C$

$= \int \frac{\sqrt{a^2-y^2}}{y}\,dy = -x + C \Longrightarrow$

$x = -\int \frac{\sqrt{a^2-y^2}}{y}\,dy + \hat{c} = a\ln\left|\frac{a}{y} + \frac{\sqrt{a^2-y^2}}{y}\right| - \sqrt{a^2-y^2} + \hat{c}$

Plug in $y(0)=14$ & $a=14 \Rightarrow \hat{c} = 0$. (Ajay)

$x = 14\ln\left|\frac{14}{y} + \frac{\sqrt{196-y^2}}{y}\right| - \sqrt{196-y^2}$

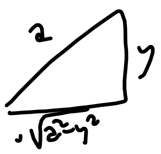
which I believe is the same as

$x = -14\ln\left|\frac{14}{y} - \frac{\sqrt{196-y^2}}{y}\right| - \sqrt{196-y^2}$

by OPTIONAL STUFF, above.

The Relationship between ln(x) and arctanh(x)

Click Here for Video



$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

$\operatorname{arctanh}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

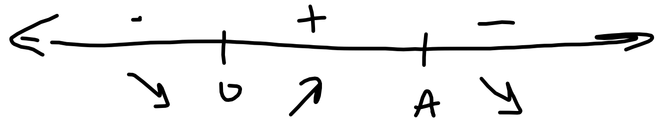
Autonomous EQ'NS. A = carrying capacity

Logistic $\frac{dP}{dt} = P(A-P)$

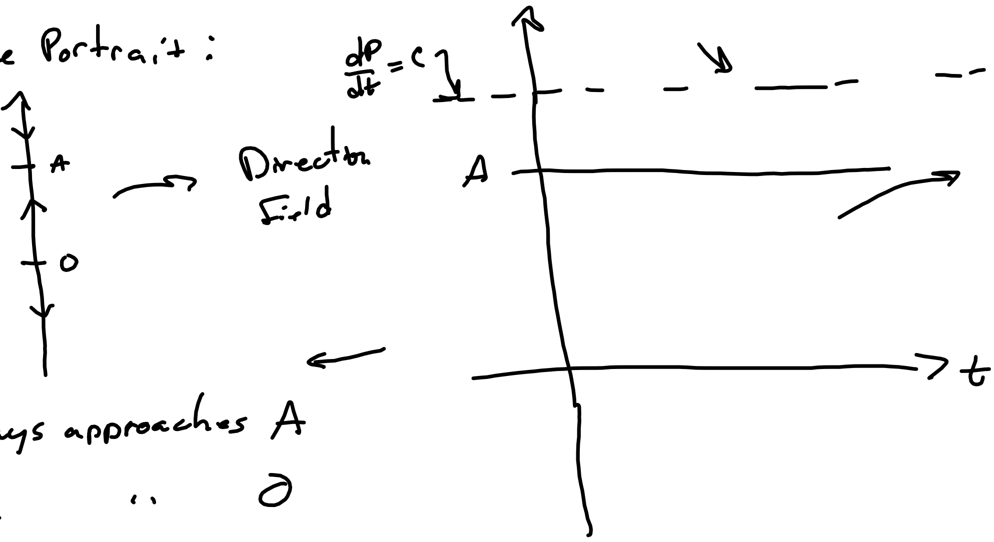
Analyze as func. of P.

$$\frac{dP}{dt} = P(A-P)$$

Sign pattern for $\frac{dP}{dt}$ as func. of P:



Phase Portrait:

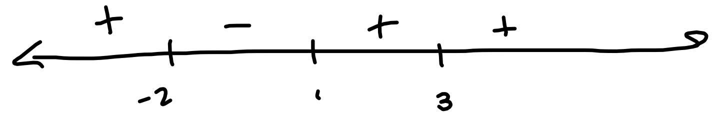


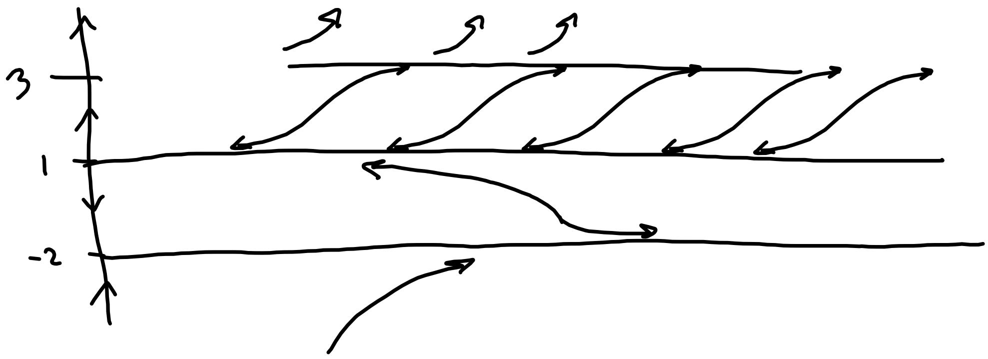
$\lim_{t\to\infty} P(t)$ always approaches A

$\lim_{t\to\infty} P(t)$ " " 0

$$\frac{dy}{dt} = f(y) = 5(y-1)(y+2)(y-3)^2$$

Automomous





CR2 # 3

$(2x+y+1)y' = 1$

Let $u = 2x+y+1$

$\Rightarrow \frac{du}{dx} = 2 + \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{du}{dx} - 2$

$u\left(\frac{du}{dx} - 2\right) = 1$

$uu' - 2u = 1$

$uu' = 2u+1$

$\frac{uu'}{2u+1} = 1$

$\frac{u\,du}{2u+1} = dx$

$$\begin{array}{r} \tfrac{1}{2} \\ 2u+1 \overline{)\, u \qquad} \\ -(u+\tfrac{1}{2}) \\ \hline -\tfrac{1}{2} \end{array}$$

$\frac{1}{2} - \frac{1}{2(2u+1)}$

$\int \frac{du}{2} - \frac{1}{2}\int \frac{1}{2u+1}\,du = \frac{1}{2}u - \frac{1}{2}\left(\frac{1}{2}\int \frac{2\,du}{2u+1}\right)$

$= \frac{1}{2}u - \frac{1}{2}\ln|2u+1| = x + C$

$= \frac{1}{2}(2x+y+1) - \frac{1}{2}\ln|2(2x+y+1)+1| = x + C$

$= x + \frac{1}{2}y + \frac{1}{2} - \frac{1}{2}\ln|4x+2y+3| = x + C$

absorb into constant

I like the Logistic

I like the conical tank draining water, where you find how fast the water level is dropping.

I want to ask a u-substitution question and I'll tell you which one it is.

Exact Differential Equation. Show it's exact and then solve it.

4.2 - A simple reduction-of-order questions.

CHEAT SHEET: One full page, front and back, is permitted.

4.3 and 4.4 stuff is all good.

Homogeneous & non homogeneous Linear ODEs w/ constant coefficients

$3y''-y'+y=0$

$3y''-y'+y=x$ or x^2 or e^x

$3m^2-m+1=0$

$3(m^2-\frac{1}{3}m+(\frac{1}{6})^2)-\frac{1}{12}+\frac{12}{12} = 3(m-\frac{1}{6})^2+\frac{11}{12}=0$

$3(\frac{1}{36})=\frac{1}{12}$

$(m-\frac{1}{6})^2=\frac{-11}{36}$

$m=\frac{1\pm i\sqrt{11}}{6}$

$\boxed{y_c=e^{\frac{1}{6}x}(c_1\cos(\frac{\sqrt{11}}{6})+c_2\sin(\frac{\sqrt{11}}{6}))}$

$3y''-y'+y=x$

$y_p=(Ax^2+Bx+C)$

$y'=2Ax+B$

$y''=2A$

$3(2A)-(2Ax+B)+Ax^2+Bx+C=x$

$\cancel{6A}-B+C=0$

$\cancel{-2A}+B\quad=1$

$\cancel{A=0}$

$\boxed{B=C=1}$

Could've done $y_p=Ax+B$. Didn't need x^2 term!

$\boxed{y_p=x+1}$

$\boxed{y=y_c+y_p}$ is the sol'n

Let r_1,r_2,r_3 be roots of C.P.
with resp. multiplicities of $m_2=7, m_1=1, m_3=2$

$\Rightarrow$ sol'n would be

$c_1e^{r_1t}+c_2e^{r_2t}+c_3te^{r_2t}+c_4t^2e^{r_2t}+\dots+c_9t^6e^{r_1t}$

$+c_{10}e^{r_3t}+c_{11}te^{r_3t}$

Order $(m_1)(m_2)(m_3)$

$=1(7)(2)=14$

$(m-1)(m-2)^2 = 0$

Gen'l sol'n $c_1e^x + c_2e^{2x} + c_3xe^{2x}$

for

$(m-1)(m^2-4m+4)$

$= m^3 - 4m^2 + 4m$

$\quad - m^2 + 4m - 4$

$m^3 - 5m^2 + 8m - 4$

$y''' - 5y'' + 8y' - 4y = 0$