4.3

Characteristic Polynomial

$y''-3y'+2y=0$

$D^2y-3Dy+2y=0$

$(D^2-3D+2)y=0$

$m^2-3m+2=0$

$\Rightarrow (m-2)(m-1)=0 \Rightarrow$

e^{2x}, e^{1x} are a fundamental set of sol'ns.

$\frac{dP}{dt}=kP$

$\Rightarrow P=C_1e^{kt}$

$P'-kP=0$

$DP-kP=0$

$(D-k)P=0$ $\quad D=k$ $\quad e^{kt}$

Special: Use Euler's Formula

for $e^{i\theta}=\cos\theta+i\sin\theta$

$\vdots$

if $m=bi$ is a sol'n of C.P.

Then e^{ibx} is a sol'n of ODE.

If coefficients of the ODE are Real, then complex roots of C.P. will occur in conjugate pairs.

x^2-4x+7

$y''-4y'+7y=0$

$m^2-4m+7=0$

m^2-4m+2^2-4+7

$=(m-2)^2+3=0$

$\Rightarrow m-2=\pm\sqrt{-3}$

$m=2\pm i\sqrt{3}$

This means $e^{(2+i\sqrt{3})x}, e^{(2-i\sqrt{3})x}$ is F. S. of sol'ns

$=e^{2x}e^{\sqrt{3}ix}, \quad e^{2x}e^{-\sqrt{3}ix}$

General sol'n :

$$c_1 e^{2x} e^{\sqrt{3} i x} + c_2 e^{2x} e^{-\sqrt{3} i x}$$

$$e^{2x} \left(e^{\sqrt{3} i x} + e^{-\sqrt{3} i x} \right)$$

$$= e^{2x} \left(\left(\cos(\sqrt{3} x) + i \sin(\sqrt{3} x)\right) + \left(\cos(-\sqrt{3} x) + i \sin(-\sqrt{3} x)\right) \right)$$

$$= e^{3x} \left(2\cos(\sqrt{3} x) \right)$$

FACT : If $m = a + bi$ is a root of C.P. then General sol'n is $e^{ax}\left(c_1 \cos(bx) + c_2 \sin(bx)\right)$

That's basically 4.3.

4.4: Handling non-homogeneous 4.3 equations.

Solve the given differential equation by undetermined coefficients.

1. $y'' - 8y' + 20y = 200x^2 - 65xe^x$

C.P.: $m^2 - 8m + 20 = m^2 - 8m + 4^2 - 16 + 20$

$= (m-4)^2 + 4 \overset{\text{SET}}{=} 0 \rightarrow$

$(m-4)^2 = -4$

$m - 4 = \pm 2i$

$m = 4 \pm 2i$

Sol'ns of homogeneous eq'n: $e^{4x}(c_1\cos(2x) + c_2\sin(2x))$

Particular Sol'n

$y_p = Ax^2 + Bx + C + e^x(Ex + F)$

"D" is reserved operator in Maple.

Sub y_p into the ODE:

$$2A + 13e^x Ex + 13e^x F - 6e^x E - 16Ax - 8B + 20Ax^2 + 20Bx + 20C$$

Equate this w/ RHS $200x^2 - 65xe^x = \text{mrhs}(x)$

I just wrote LHS − RHS = 0 & collected terms. This gives.

$$2A + 13e^x Ex + 13e^x F - 6e^x E - 16Ax - 8B + 20Ax^2 + 20Bx + 20C - 200x^2 + 65xe^x = 0$$

This can be solved by back-sub, or...

$eqn1 := 2A - 8B + 20C = 0$

$eqn2 := 13E + 65 = 0$

$eqn3 := 13F - 6E = 0$

$eqn4 := -16A + 20B = 0$

$eqn5 := 20A - 200 = 0$

$$\begin{aligned} 2A - 8B + 20C &= 0 \\ 13E + 65 &= 0 \\ -6E \quad 13F & \\ -16A + 20B &= 0 \\ 20A \quad -200 &= 0 \end{aligned}$$

$$\begin{aligned} 2A - 8B + 20C &= 0 \\ 13E + 65 &= 0 \\ -6E + 13F &= 0 \\ -16A + 20B &= 0 \\ 20A \quad -200 &= 0 \end{aligned}$$

$$\begin{array}{ccccc|c} A & B & C & E & F & \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 2 & -8 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13 & 0 & -65 \\ -16 & 20 & 0 & 0 & 0 & 0 \\ 20 & 0 & 0 & 0 & 0 & 200 \\ 0 & 0 & 0 & -6 & 13 & 0 \end{array}\right]$$

Forgot Row 3

$mymatrix := \langle 2, -8, 20, 0, 0, 0; 0, 0, 0, 13, 0, -65; -16, 20, 0, 0, 0, 0; 0, 0, 0, -6, 13, 0; 20, 0, 0, 0, 0, 200 \rangle$

$$mymatrix := \begin{bmatrix} 2 & -8 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13 & 0 & -65 \\ -16 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 & 13 & 0 \\ 20 & 0 & 0 & 0 & 0 & 200 \end{bmatrix}$$

$ReducedRowEchelonForm(mymatrix)$

$A = 10$
$B = 8$
$C = \frac{11}{5}$
$E = -5$
$F = -\frac{30}{13}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 0 & \frac{11}{5} \\ 0 & 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 0 & 1 & -\frac{30}{13} \end{bmatrix}$$

CHAPTER 3 WRITTEN:

$$\frac{dT}{dt} = k\left(T^4 - T_m^4\right)$$

$$\frac{dT}{T^4 - T_m^4}$$

Use Wolfram Alpha
I tried doing Partial fractions.

$$\frac{1}{T^4 - T_m^4}$$

$$\frac{dT}{dt} = k\left(T^4 - T_m^4\right)$$

$$= k\left(\left(T_m + T - T_m\right)^4 - T_m^4\right)$$

$$= k\left(\left(T_m\left(1 + \frac{T - T_m}{T_m}\right)\right)^4 - T_m^4\right)$$

$$= k\left(T_m^4\left(1 + \frac{T - T_m}{T_m}\right)^4 - T_m^4\right)$$

$$= kT_m^4\left(\left(1 + \frac{T - T_m}{T_m}\right)^4 - 1\right)$$

1
1 1
1 2 1
1 3 3 1
$\boxed{1\ 4\ 6\ 4\ 1}$

Binomial Theorem Review

$$(a+b)^4 = \sum_{k=0}^{4} \binom{4}{k} a^{4-k} b^k = 1a^4 + 4a^3b^1 + 6a^2b^2 + 4ab^3 + b^4$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$ n is a positive integer

$\binom{n}{k} = {}_nC_k$ = "n choose k" = # of ways you can choose k things from n things. = the # of subsets of a set of n elements that contain k elements.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)(n-2)\cdots(n-k+1)\cancel{(n-k)\cdots(3)(2)(1)}}{\left(k(k-1)\cdots(3)(2)(1)\right)\cancel{\left((n-k)(n-(k+1))(n-(k+2))\cdots(2)\right.}(1)\Big)}$$

$$\binom{5}{3} = \frac{5!}{3!\,2!} = \frac{5\cdot 4}{2!} = 10$$

$$\binom{4}{2} = \frac{4\cdot 3}{2} = 6$$

$\{a,b,c,d\}$

$\{a,b\}, \{a,c\}, \{a,d\}\ \{b,c\}, \{b,d\}, \{c,d\}$

$\{a,b\} = \{b,a\}$

$$kT_m^4\left(\left(1+\frac{T-T_m}{T_m}\right)^4 - 1\right)$$

$$(1)(1)^4 + 4(1)^3\left(\frac{T-T_m}{T_m}\right) + 6(1)^2\left(\frac{T-T_m}{T_m}\right)^2 + 4(1)^1\left(\frac{T-T_m}{T_m}\right)^3 + \left(\frac{T-T_m}{T_m}\right)^4$$

$$= \underbrace{1 + 4\left(\frac{T-T_m}{T_m}\right) + 6\left(\frac{T-T_m}{T_m}\right)^2}_{\text{1st 3 terms}} + \underbrace{4\left(\frac{T-T_m}{T_m}\right)^3 + \left(\frac{T-T_m}{T_m}\right)^4}_{\text{small}}$$

$T - T_m$ "small"

Throw out the $\left(\frac{T-T_m}{T_m}\right)^2$ term

$$\frac{dT}{dt} \approx kT_m^4\left(1 + 4\frac{T-T_m}{T_m} - 1\right) = 4kT_m^4\left(\frac{T-T_m}{T_m}\right)$$

$$= 4kT_m^3(T - T_m)$$

So it <u>IS</u> Newton's Law

with ITS Konstant $K = 4kT_m^3$