

Load Packages we'll be using. We need "VectorCalculus" for the Wronskian command and we'll need "LinearAlgebra" for a bunch of commands.

with(VectorCalculus) :

with(LinearAlgebra) :

To enter a matrix, row by row, separate the entries by commas and the rows with a semi-colon:

$A := \langle 1, 1 ; 4, 1 \rangle$

$$A := \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \quad (1)$$

Clobber the determinant with **Determinant** command:

Determinant(A)

$$-3 \quad (2)$$

Find the inverse matrix of A with the **MatrixInverse** command:

MatrixInverse(A)

$$\begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{4}{3} & -\frac{1}{3} \end{bmatrix} \quad (3)$$

Check that they're inverses. For matrix multiplication, use a period. The Maple reformatted it so it's hard to tell.

MatrixInverse(A) • A

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (4)$$

Enter a column vector:

$bbar := \langle 1; 3 \rangle$

$$bbar := \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad (5)$$

$xbar := \langle x1; x2 \rangle$

$$xbar := \begin{bmatrix} x1 \\ x2 \end{bmatrix} \quad (6)$$

Solve the equation $Axbar = bbar$

MatrixInverse(A) • bbar

$$\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} \quad (7)$$

$B := \langle 1, 2, 3 ; 4, 5, 6; 7, 8, 10 \rangle$

$$B := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix} \quad (8)$$

We can compute the determinant of a matrix with the **Determinant** command:

$$\text{Determinant}(B) \quad -3 \quad (9)$$

The **Wronskian** command is a particular determinant of the Wronski matrix. The 'determinant' in quotes returns the value of the determinant after showing you the uncomputed determinant. Note: 3 functions entered -> 3x3 determinant.

$$\text{Wronskian}([\exp(t), \sin(t), t], t, 'determinant') \quad \begin{bmatrix} e^t & \sin(t) & t \\ e^t & \cos(t) & 1 \\ e^t & -\sin(t) & 0 \end{bmatrix}, -e^t t \sin(t) - e^t \cos(t) t + 2 e^t \sin(t) \quad (10)$$

Two functions entered -> 2x2 determinant:

$$\text{Wronskian}([\exp(t), \sin(t)], t, 'determinant') \quad \begin{bmatrix} e^t & \sin(t) \\ e^t & \cos(t) \end{bmatrix}, e^t \cos(t) - e^t \sin(t) \quad (11)$$

The two functions are linearly independent if and only if the Wronskian (determinant) is *never* zero for **all values of t !!!**

$$\text{solve}(e^t \cos(t) - e^t \sin(t) = 0, t) \quad \frac{\pi}{4} \quad (12)$$

So there's a value of t that makes the determinant zero. This means that the two functions aren't linearly dependent!

However, you *can* make them linearly independent on any interval not containing $\pi/4$ or any other value of t such that $\tan(t) = 1$.

$$\text{Wronskian}([\exp(x), \exp(-x), \sinh(x)], x, 'determinant')$$

$$\begin{bmatrix} e^x & e^{-x} & \sinh(x) \\ e^x & -e^{-x} & \cosh(x) \\ e^x & e^{-x} & \sinh(x) \end{bmatrix}, 0 \tag{13}$$

Example 7

Wronskian([exp(3·x), exp(−3·x)], x, 'determinant')

$$\begin{bmatrix} e^{3x} & e^{-3x} \\ 3e^{3x} & -3e^{-3x} \end{bmatrix}, -6e^{3x}e^{-3x} \tag{14}$$

simplify(−6 e^{3x} e^{−3x})

$$-6 \tag{15}$$