$N_0 = 600$

$N(1) = 1200$

$K = \frac{a}{b} =$ carrying capacity $= 60000 = \lim_{t \to \infty} N(t)$

$$N(t) = \frac{aN_0}{bN_0 + (a - bN_0)e^{-at}}$$

$$N_0 = \frac{aN_0}{bN_0 + (a - bN_0)e^{-a(0)}} = \frac{aN_0}{bN_0 + (a - bN_0)} \quad \therefore \checkmark$$

$$\left(\begin{array}{l} \frac{a}{b} = 60000 \\ a = 60000b \end{array} \right) \quad = \frac{60000b\,N_0}{bN_0 + (60000b - bN_0)e^{-at}}$$

$$= \frac{60000N_0}{N_0 + (60000 - N_0)e^{-60000bt}}$$

$$N(1) = \frac{60000N_0}{}$$

$$\frac{K}{1 + be^{-at}} \qquad \frac{60000}{1 + be^{-at}}$$

$N_0 = 600$

$$= \frac{60000}{1+b} = 600$$

$$60000 = 600 + 600b$$

$$59400 = 600b$$

$$\frac{59400}{600} = b = \frac{594}{6} = \frac{297}{3} = 99$$

$$N(1) = \frac{K}{1 + be^{-a}} \implies N(1)(1 + be^{-a}) = K$$

$$N(1) + bN(1)be^{-a} = K$$

$$N(1)be^{-a} = K \quad N(1)$$

$$e^{-a} = \frac{K - N(1)}{bN(1)} = \frac{60000 - 1200}{99(1200)}$$

$$= \frac{58800}{99(1200)} = \frac{588}{99(12)} = \frac{294}{99(6)} = \frac{147}{99(3)}$$

$$= \frac{49}{33(3)} = \frac{49}{99} \longrightarrow \quad \frac{\frac{1254}{2}}{588}.$$

$$-a = \ln\left(\frac{49}{99}\right)$$

$$\implies \boxed{a = \ln\left(\frac{99}{49}\right)}$$

$$N(t) = \frac{2N_0}{bN_0 + (2 - bN_0)e^{-2t}}$$

$N_0 = 600$

$N(1) = 1200$

$$= \frac{60000bN_0}{bN_0 + (60000b - bN_0)e^{-2t}}$$

$$= \frac{N_0}{N_0 + 59400e^{-2t}}$$

$$= \frac{600}{600 + 59400e^{-2t}} = \frac{6}{6 + 594e^{-2t}}$$

e

A model for the population $P(t)$ in a suburb of a large city is given by the initial-value problem

2.

$$\frac{dP}{dt} = P(10^{-1} - 10^{-7}P), \qquad P(0) = 4{,}000,$$

$$\frac{dP}{dt} = kP(a - bP)$$

where t is measured in months.

Find the population P of the suburb at time t.

$P(t) =$ $\boxed{\dfrac{400}{0.0004 + 0.0996e^{-0.1t}}}$

$$\int \frac{dP}{P(10^{-1} - 10^{-7}P)} = \int dt$$

What is the limiting value of the population?

1,000,000

At what time (in months) will the population be equal to one-half of this limiting value? (Round your answer to one decimal place.)

55.2 months

3. Two chemicals A and B are combined to form a chemical C. The rate, or velocity, of the reaction is proportional to the product of the instantaneous amounts of A and B not converted to chemical C. Initially, there are 40 grams of A and 50 grams of B, and for each gram of B, 2 grams of A is used. It is observed that 25 grams of C is formed in 8 minutes. How much (in grams) is formed in 16 minutes? (Round your answer to one decimal place.)

[ ] ✗ 38.1 grams

What is the limiting amount (in grams) of C after a long time?

[ ] ✗ 60 grams

How much (in grams) of chemicals A and B remains after a long time?

A [ ] ✗ 0 grams

B [ ] ✗ 30 grams

Let

$A = \#$ of grams of chem. A remaining

$B =$ " " " " B "

Let $x = x(t)$ be the amt (in grams) of Product C that exists.

x is M Parts A, N Parts B

$$A = a - \frac{M}{M+N}x(t)$$

$$\& \quad B = b - \frac{N}{M+N}x(t)$$

Then $\frac{dx}{dt} = k\left(a - \frac{M}{M+N}x\right)\left(b - \frac{N}{M+N}x\right)$ SEPARABLE

$= k\left(\frac{MN}{(M+N)^2}\right)(\alpha - x)(\beta - x)$, where $\alpha = a\left(\frac{M+N}{M}\right)$

$\& \quad \beta = b\left(\frac{M+N}{N}\right)$

$\rightarrow \int \frac{dx}{(\alpha - x)(\beta - x)} = \int dt$, etc

Do convert$(f(x)$, fullparfrac, $x)$

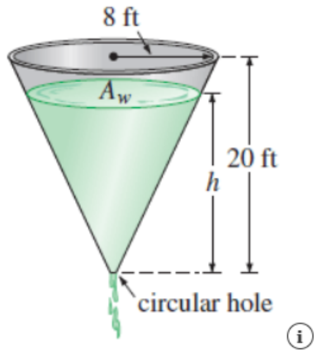
4. Suppose water is leaking from a tank through a circular hole of area A_h at its bottom. When water leaks through a hole, friction and contraction of the stream near the hole reduce the volume of water leaving the tank per second to $cA_h\sqrt{2gh}$, where c $(0 < c < 1)$ is an empirical constant.

A tank in the form of a right-circular cone standing on end, vertex down, is leaking water through a circular hole in its bottom. (Assume the removed apex of the cone is of negligible height and volume.)

(a) Suppose the tank is 20 feet high and has radius 8 feet and the circular hole has radius 2 inches. The differential equation governing the height h in feet of water leaking from a tank after t seconds is

$$\frac{dh}{dt} = -\frac{5}{6h^{3/2}}.$$

In this model, friction and contraction of the water at the hole are taken into account with $c = 0.6$, and g is taken to be 32 ft/s^2. See the figure below.



Tank is initially full →
$h(0) = 20$

$-\int 6h^{3/2}\,dh = \int 5\,dt$

$-6\left(\frac{2}{5}\right)h^{5/2} = 5t + \hat{c}$

$h^{5/2} = -\frac{5}{12}(5t + \hat{c}) = -\frac{25}{12}t - \frac{5}{12}\hat{c}$

$h = \left(-\frac{25}{12}t - \frac{5}{12}\hat{c}\right)^{2/5}$

$h(0) = \left(-\frac{5}{12}\hat{c}\right)^{2/5} = 20$ →

$-\frac{5}{12}\hat{c} = 20^{5/2} = 400\sqrt{20}$

$\hat{c} = -\frac{12}{5}(800\sqrt{5})$

→ $h(t) = \left(-\frac{25}{12}t - \frac{5}{12}\left(-\frac{12}{5}\cdot 800\sqrt{5}\right)\right)^{2/5}$

Solve the initial value problem that assumes the tank is initially full.

$h(t) =$ ✗ $\left(800\sqrt{5} - \frac{25t}{12}\right)^{2/5}$

If the tank is initially full, how long (in minutes) will it take the tank to empty? (Round your answer to two decimal places.)

✗ 14.31 minutes

SET $h(t) = 0$ $\quad h(t) = \left(800\sqrt{5} - \frac{25}{12}t\right)^{2/5}$

→ $\frac{25}{12}t = 800\sqrt{5}$

$t = (800\sqrt{5})\left(\frac{12}{25}\right) = 32(12)\sqrt{5} \approx 384\sqrt{5}$

$\approx$ 858.6501032 sec

$\approx \boxed{14.31\ \text{min}}$

Suppose the tank has a ve of 60° and the circular hole has radius 3 inches. Determine ~~the differential~~ equation governing the height h of water. Use $c = 0.6$ and $g = 32$ ft/s^2.

$\frac{dh}{dt} =$ ✗ $-\frac{9}{10h^{3/2}}$

32
25
160
640
800

32
12
64
320
384

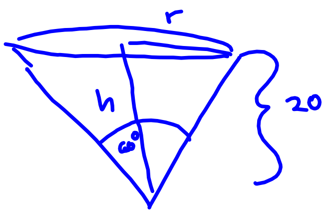
Solve the initial value problem that assumes the height of the water is initially 10 feet.

$h(t) =$ ✗ $\left(100\sqrt{10} - \frac{9t}{4}\right)^{2/5}$

If the height of the water is initially 10 feet, how long (in minutes) will it take the tank to empty? (Round your ansı to two decimal places.)

✗ 2.34 minutes

$cA_h\sqrt{2gh}$

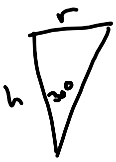


$\frac{r}{h} = \tan 30°$

$r = \frac{h}{\sqrt{3}}$

§3.2 #4 vertex @ 60°.

Now $V = \frac{1}{3}\pi r^2 h$ (Volume of cone)



$r = h\tan 30° = \frac{h}{\sqrt{3}}$

$$= \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h$$

$$= \frac{1}{3}\pi\frac{h^3}{3} = \frac{1}{9}\pi h^3$$

Update to 3.2 #4: $\frac{dV}{dt} = -cA_p\sqrt{2gh}$

$$\Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi h^2\frac{dh}{dt} = -cA_p\sqrt{2gh}$$

$$\frac{dh}{dt}h^2 = \frac{-3cA_p}{\pi}\sqrt{2gh} = \frac{-3\left(\frac{6}{10}\right)\pi\left(\frac{1}{6}\right)^2}{\pi}\sqrt{2gh}$$

3'' pipe! $\frac{3}{12} = \frac{1}{4}$!

$$= -\frac{9}{5}\left(\frac{1}{16}\right)8h^{\frac{1}{2}} = -\frac{9}{10}h^{\frac{1}{2}} \checkmark$$

$$\Rightarrow \boxed{\frac{dh}{dt} = -\frac{9}{10}h^{-\frac{3}{2}}}$$

$$\Rightarrow \int h^{\frac{3}{2}}\,dh = \int \frac{9}{10}\,dt$$

$$\frac{2}{5}h^{\frac{5}{2}} = -\frac{9}{10}t + C$$

$$h^{\frac{5}{2}} = -\frac{9}{10}\cdot\frac{5}{2}t + \frac{5}{2}C$$

$$h(t) = \left(-\frac{9}{4}t + \frac{5}{2}C\right)^{\frac{2}{5}}$$

h(

GOOD STUFF