Are you using the solutions for that first Written Work?

Your score on WebAssign for the Chapter Review doesn't count. You need to scan your written work and upload it to Assignments on D2L.

Checking due date (Should be Friday, 11:59 pm.)

We have Written Work Assignments built on D2L.

Click Here to open up the Course Shell and look at your new Assignments link on the Main Navbar.

They depart somewhat from what's in the current Course Schedule, but they're more reasonable. I'll be editing that Course Schedule all semester, and tweaking due dates on things.

I can already see us slopping into that first week of optional work. I want to stay as close to the planned schedule (crack the whip) early as I can, so we can back off a little at the end. That always works better, with so many young professors out there realizing they have 4 chapters to cover in the last week!

First-Order differential equation in normal form:

Eq'n (2) from §1.2

Solve: $\frac{dy}{dx} = f(x, y)$

Subject to: $y(x_0) = y_0$ (2)

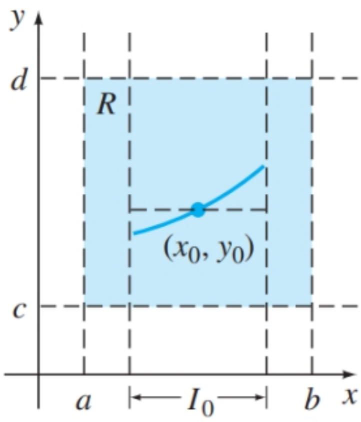
Recall: (From reading, not lecture)

Theorem 1.2.1 Existence of a Unique Solution

Let R be a rectangular region in the xy-plane defined by $a \le x \le b$, $c \le y \le d$ that contains the point (x_0, y_0) in its interior. If $f(x, y)$ and $\partial f/\partial y$ are continuous on R, then there exists some interval I_0: $(x_0 - h, x_0 + h)$, $h > 0$, contained in $[a, b]$, and a unique function $y(x)$, defined on I_0, that is a solution of the initial-value problem (2).

$x \leftarrow (2, 6)$

y



Existence is guaranteed by continuity of the derivative. Easy to check.

Uniqueness is guaranteed by requiring the solution pass through the point (x_0, y_0).

Recall "Normal Form"

Get the dy/dx all by itself on one side.

"Lineal Element" is just a short segment of the tangent line at a given point on the solution curve.

We're leading up to Euler's Method.

It's a required topic in Calculus II, so you should have some familiarity with it and have built at least one direction field by hand..

I'm still waiting to hear back from the Maple people about purchasing Maple for you guys. Short term, we can do direction (slope) fields, using Desmos:

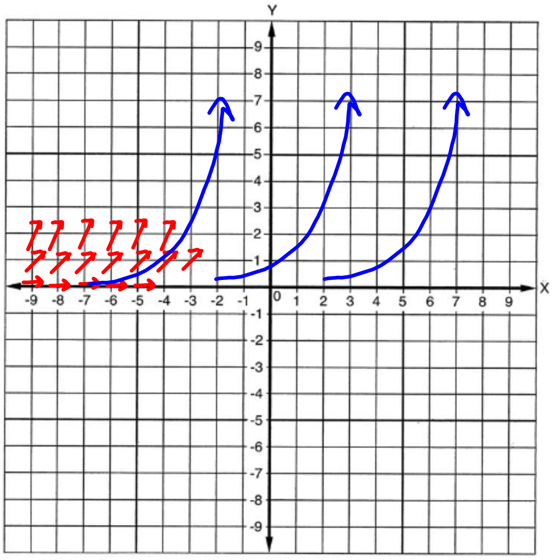
Are you still on "temporary" access on WebAssign?

Section 2.1 - Direction Fields (Slope Fields) for 1st-order ODE in normal form.

We plot solutions without knowing what the formula is or if there even is a clean formulaic solution.

$y' = f(x,y)$ (1) in §2.1

How about $y' = y$?



$y' - y = 0$

$(D-1)y = 0 \to ce^{+x}$

↗ - Lineal Element

This equation is AUTONOMOUS - No x-variable in the

$f(x,y) = y = f(y)$

For an autonomous equation, the solutions are just horizontal shifts of one another.

Direction Field If we systematically evaluate f over a rectangular grid of points in the xy-plane and draw a line element at each point (x, y) of the grid with slope $f(x, y)$, then the collection of all these line elements is called a **direction field** or a **slope field** of the differential equation $dy/dx = f(x, y)$. Visually, the direction field suggests the appearance or shape of a family of solution curves of the differential equation, and consequently, it may be possible to see at a glance certain qualitative aspects of the solutions—regions in the plane, for example, in which a solution exhibits an unusual behavior.

Maple Interlude:

Your book makes a number of qualitative remarks about direction fields. Some of them are actually helpful. I will try to make some good qualitative remarks about the equations we're looking at in Maple.

Eq'n 1: $y' = -y$

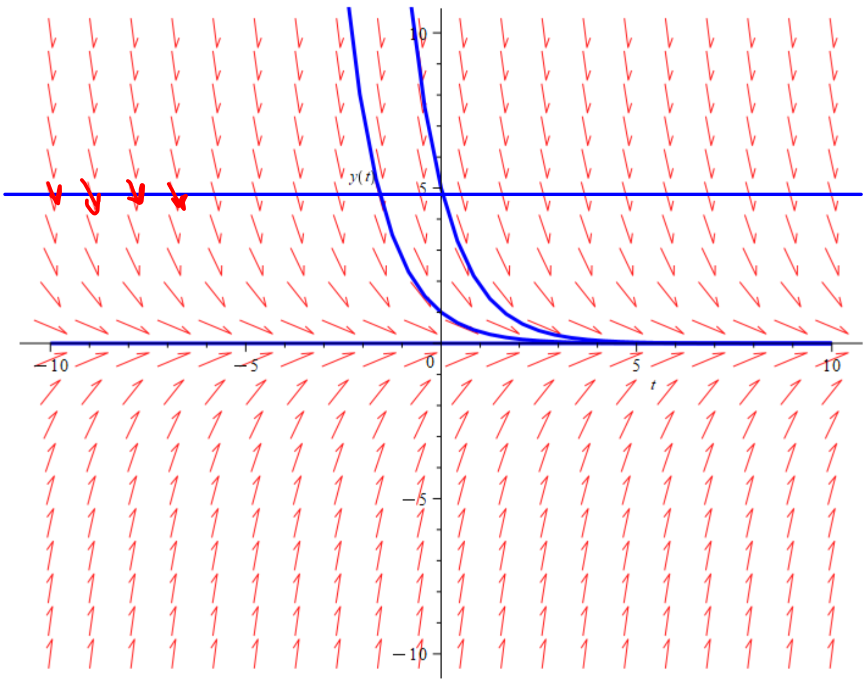
Eq'n 2: $y' = t + y$ ($y' = x+y$ would be the same)

Also did Example 3.

$y' = -y$

myeqn := diff(y(t), t) = -y(t);

mydirectionfield := DEplot(myeqn, y(t), t = -10 .. 10, y = -10 .. 10, [y(0) = 1, y(0) = 0, y(0) = 5], linecolor = blue);



Desmos: Search for "Direction fields on Desmos"

https://www.desmos.com/calculator/p7vd3cdmei

Autonomous 1st-Order ODEs: If the right-hand side of the equation (in normal form) is independent of the independent variable , in other words, if you don't see the independent variable appearing on the RHS, then the equation is autonomous.

$y' = f(y)$ (Not $y' = f(x,y)$) EQN (2) in §2.1

$$\frac{dA}{dt} = kA, \qquad \frac{dx}{dt} = kx(n + 1 - x), \qquad \frac{dT}{dt} = k(T - T_m), \qquad \frac{dA}{dt} = 6 - \frac{1}{100}A,$$

where k, n, and T_m are constants, shows that each equation is time independent.

1. uninhibited pop
2. Disease Spread
3. Newton's Law of Cooling
4. Kirchhoff's Laws (Voltage drop)

Critical Points: The zeros of the function *f* on the RHS of EQN (2).

Critical Points are also called "equilibrium points" or "stationary points."

If we substitute y = constant into the RHS, this means both sides of EQN (2) are zero.

This means that if c is a critical point of (2), then $y = c$ is a constant solution of the autonomous differential equation.

Example 3 - Looks a lot like a logistic population growth model.

$$\boxed{\frac{dP}{dt} = P(a-bP)} = aP\left(1-\frac{P}{\left(\frac{a}{b}\right)}\right)$$

Tweaking it to look like logistic population growth model.

LOGISTIC:

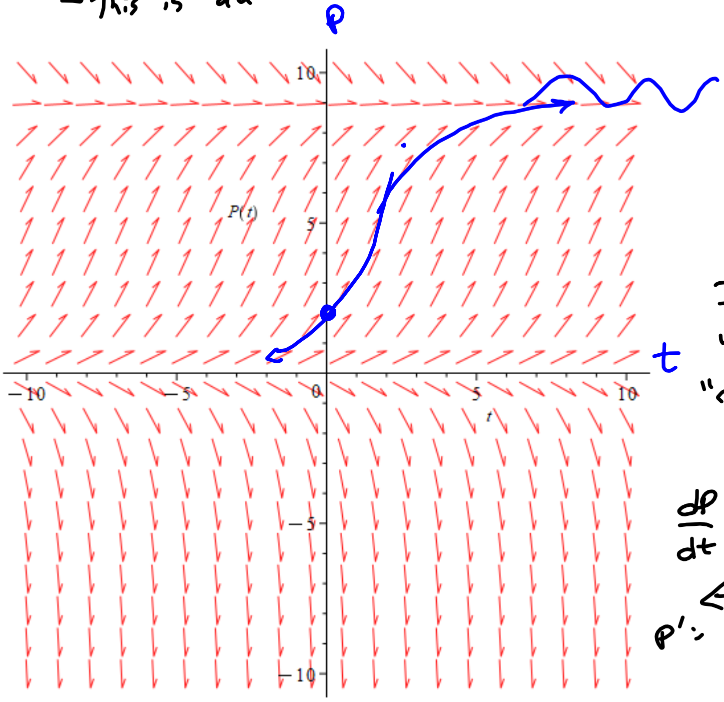
$$\frac{dP}{dt} = rP\left(1-\frac{P}{K}\right) \stackrel{\text{SET}}{=} 0$$ for CRITICAL points.

r = "intrinsic Growth RATE"

K = carrying capacity

This is autonomous.

Phase portrait is a vertical rendering of a sign pattern for the derivative, with arrows indicating positive or negative (slopes).



$P(a-bP) \stackrel{\text{SET}}{=} 0 \Rightarrow$

$P=0$ OR $a-bP=0 \Rightarrow$

$a=bP \Rightarrow$

$P=\frac{a}{b}$

In this example, I'm using $a=1, b=\frac{1}{9}$ to get "carrying capacity" of 9.

Sign Pattern for

$$\frac{dP}{dt} = P(a-bP) = -bP^2+aP$$

P': $-$ ↘ 0 $+$ ↗ $\frac{a}{b}$ $-$ ↘

"One-Dimensional Phase Portrait"

P decreasing for $P \in \left(\frac{a}{b}, \infty\right) \cup (-\infty, 0)$

& P increasing $\forall$ $P \in \left(0, \frac{a}{b}\right)$.

If we partition the P axis into 3 parts, as shown, we see that Theorem 1.2.1 gives us uniqueness of solutions in horizontal strips corresponding to each subinterval.

In other words,

whatever (t_0, P_0) we're given as an initial value, solutions will be "trapped" in one of the horizontal strips. For logistic population models, we always end up in the P-interval $\left(0, \frac{a}{b}\right)$ and the MODEL solution has t-domain $(-\infty, \infty)$.

Of course, we don't fool around with negative time in the real world, or negative population.

Continuity of the right-hand-side means the derivative doesn't change sign.

Boundedness of the region means that all solutions will approach the equilibrium solution(s).

The book states this fact without proof:

- If $y(x)$ is *bounded above* by a critical point c_1 (as in subregion R_1 where $y(x) < c_1$ for all x), then the graph of $y(x)$ must approach the graph of the equilibrium solution $y(x) = c_1$ either as $x \to \infty$ or as $x \to -\infty$. If $y(x)$ is *bounded*—that is, bounded above and below by two consecutive critical points (as in subregion R_2 where $c_1 < y(x) < c_2$ for all x)—then the graph of $y(x)$ must approach the graphs of the equilibrium solutions $y(x) = c_1$ and $y(x) = c_2$, one as $x \to \infty$ and the other as $x \to -\infty$. If $y(x)$ is *bounded below* by a critical point (as in subregion R_3 where $c_2 < y(x)$ for all x), then the graph of $y(x)$ must approach the graph of the equilibrium solution $y(x) = c_2$ either as $x \to \infty$ or as $x \to -\infty$. See Problem 34 in Exercises 2.1.

Basically, it's saying that solutions approach their critical points in whichever subinterval of the phase portrait you're in, and the solution will never crosee one of its critical points.

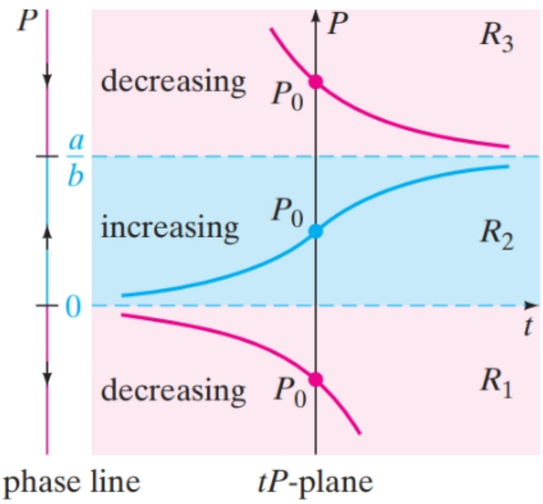


Figure 2.1.7 Phase portrait and solution curves in Example 4

Example 5

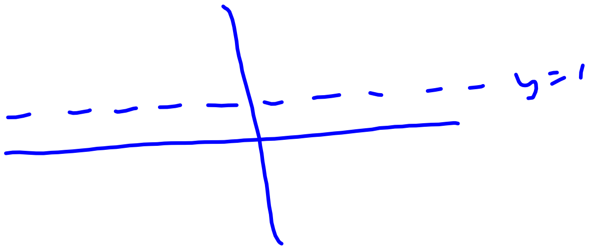
$\frac{dy}{dx} = (y-1)^2$ is autonomous

$(y-1)^2 = 0 \Rightarrow$

$y=1$ is c.p. of the ODE.

soln's live above or below.

since $(y-1)^2 \geq 0$, slope is positive away from $y=1$.



Let's look for solns

$y' = (y-1)^2$

$\frac{dy}{dx} = (y-1)^2 \Rightarrow$

$\frac{dy}{(y-1)^2} = dx \Rightarrow$

$(y-1)^{-2}dy = dx$

$\int (y-1)^{-2}dy = \int dx = x+c \Rightarrow$

$-(y-1)^{-1} = x+c \Rightarrow$

$\frac{1}{x+c} = -(y-1) \Rightarrow -y+1 = \frac{1}{x+c} \Rightarrow$

$-y = -1 + \frac{1}{x+c} \Rightarrow y = 1 - \frac{1}{x+c}$

We haven't done any "solution techniques" to speak of, yet. This is just applying Calculus skills to finding *y*.

This *c* can be any real number. We can solve for *c*, given an initial value.

If $y(0)=1$:

$y\equiv 1$ is ! sol'n

We're looking for continuous & differentiable solutions, and these all have a vertical asymptote @ $x=-c = -\frac{1}{2}$ in this case

Since $y(0)=-1$ is to the right of $x=-c=-\frac{1}{2}$, we have a domain of $(-\frac{1}{2},\infty)$ & we ditch the left half.

If $y(0)=-1$

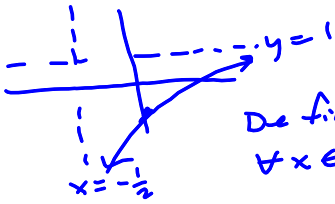
$1-\frac{1}{0+c} = 1-\frac{1}{c} = -1 \rightarrow$

$-\frac{1}{c} = -2 \rightarrow$

$c=\frac{1}{2}$

$y = 1-\frac{1}{x+\frac{1}{2}}$ has V.A. $x=-\frac{1}{2}$

so the sol'n is



Defined $\forall x \in (-\frac{1}{2},\infty)$

Attractors and Repellers:

Sinks and Sources

Sink : Asymptotic approach to c.p. STABLE EQUILIBRIUM

Source : Arrows Point away from c.p. UNSTABLE EQ.

Equilibrium

Semi-Stable: Arrows in & arrows out.

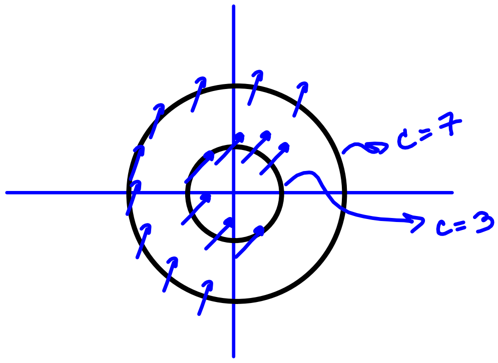
Classifying Critical Points as stable/unstable. (Arrows toward or away in phase portrait?)

Note: All solutions to autonomous solutions are horizontal translations of one another.

This is because the slopes are constant at any fixed *y*-value all the way all the way across the direction field.

So if $y = g(x)$ is a solution, then so it $y = g(x - k)$ for any k.

$\frac{dy}{dx} = \sqrt{x^2+y^2}$



Isoclines are circles

$\sqrt{x^2+y^2} = c \implies$

$x^2+y^2 = c^2$ is a circle of radius $r=c$