Falling Body

won't be relevant

$h = -\frac{1}{2}gt^2 + v_0 t + h_0$

$\frac{dh}{dt} = v = -gt + v_0 = -gt$

h = height in ft
$g = -32 \, ft/s^2$
t = time in seconds
$h_0 = h(0)$
$v_0 = v(0)$

Derivation

Potential Energy $= mgh = \frac{1}{2}mv^2$
= Kinetic Energy

$\Rightarrow gh = \frac{1}{2}v^2$

$\Rightarrow v = \sqrt{2gh}$ = velocity $\left(\frac{ft}{s}\right)$

Area of hole is A_H $\Rightarrow$

V = volume $\overset{?}{=} \pi r^2 h$ Right circular cylinder

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$ r is fixed

A_H = Area of hole.

$\frac{dV}{dt} = A_H v = -A_h\sqrt{2gh}$

If area of the top of the water is constant,

say A_w, then

$\frac{dV}{dt} = A_w \frac{dh}{dt}$

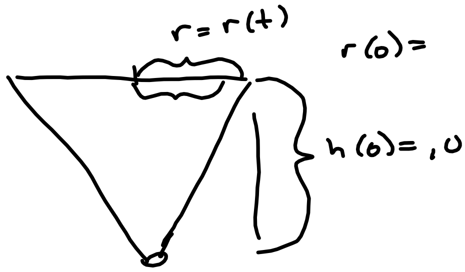
$\frac{dV}{dt} = -A_h\sqrt{2gh} = A_w \frac{dh}{dt} \Rightarrow$

$\frac{dh}{dt} = -\frac{A_h}{A_w}\sqrt{2gh}$

This is model from example.

What happens when the area at the top of the water is a function of time? $A_w = A(t)$

$$\frac{dh}{dt} = -\frac{A_H}{A_w}\sqrt{2gh} = -\frac{A_H}{A_w}\sqrt{2gh}$$



$r(0) =$

$$\frac{r}{h} = \frac{4}{10} = \frac{2}{5}$$

$$\Longrightarrow r(t) = \frac{2}{5}h(t)$$

$$A_w = A_w(t) = \pi r^2 = \pi\left(\frac{2}{5}h\right)^2 = \frac{4\pi h^2}{25}$$

$$\frac{dh}{dt} = -\frac{A_H}{A_w}\sqrt{2gh} = -\frac{A_H}{\frac{4\pi h^2}{25}}\sqrt{2gh}$$

$$= \frac{-25A_H}{4\pi h^2}\sqrt{2g}\, h^{\frac{1}{2}} = -\frac{25A_H}{4\pi}\sqrt{2g}\, h^{-\frac{3}{2}}$$