Section 1.2 - Initial Value Problems

Definee nth-order initial value problem (IVP)

Solve $$\frac{d^n y}{dx^n} = f(x, y, y', \dots, y^{(n-1)})$$

$$y^{(n)} = f(x, y^{(0)}, y^{(1)}, y^{(2)}, \dots, y^{(n-1)})$$

Subject to

$$y^{(k)}(x_0) = y_0^{(k)}, \quad k = 0, 1, \dots, n-1$$

$y^{(0)}(x_0) = y(x_0) = y_0$, $y'(x_0) = y^{(1)}(x_0) = y_1$, $y''(x_0) = y^{(2)}(x_0) = y_2$, …

, …, $y^{(n-1)}(x_0) = y_{n-1}$, where

$y_0, \dots, y_{n-1}$ are arbitrary constants.

$n^{\underline{th}}$ order, we expect n constants or parameters $c_1, \dots, c_n$ that are uniquely (we hope!) determined by the n constants $y_0, y_1, \dots, y_{n-1}$.

1st- and 2nd-order IVPs are easier to visualize than higher-order IVPs.

E1 (a) 1^{st}-order $y' = y$, s.t. $y(0) = 3$

Solns : $y = \phi = ce^t$ $c = 3$

(b) 1^{st} order $y' = y$, s.t. $y(1) = -2$ Fig 1.2.3 , pg 16

$ce^1 = -2 \Rightarrow$

$c = -\frac{2}{e}$ $-\frac{2}{e}e^t = 2e^{-1}e^t = 2e^{t-1}$

1^{st} ORDER : $y' = F(x, y)$

s.t. $y(0) = y_0$

2^{nd} ORDER : $y'' = F(x, y, y')$

$y(1) = 7, y'(1) = 8$

⋮

4^{th} ORDER $y'''' = F(x, y, y, y'', y''') = y^{(4)}$