Section 4.3

Find the general solution of the given second-order differential equation.

3 $2y'' - 5y' + 6y = 0$

$2m^2 - 5m + 6 = 0$

$a = 2$, $b = -5$, $c = 6$

$b^2 - 4ac = 5^2 - 4(2)(6) = 25 - 48 = -23$

$x = \frac{5 \pm i\sqrt{23}}{4} = \frac{5}{4} \pm i\frac{\sqrt{23}}{4}$

So $y = e^{\frac{5}{4}x}\left(c_1 \cos\left(\frac{\sqrt{23}}{4}x\right) + c_2 \sin\left(\frac{\sqrt{23}}{4}x\right)\right)$

Repeated Roots

Consider the following higher-order differential equation.

5. $y^{(4)} - 2y'' + y = 0$

Find all the roots of the auxiliary equation. (Enter your answer as a comma-separated list.)

$m^4 - 2m^2 + 1 = 0$

$(m^2-1)^2 = (m-1)^2(m+1)^2$

$m=1$, mult. of 2.

$m=-1$, " " 2.

Gen'l Sol'n: $c_1e^x + c_2xe^x + c_3e^{-x} + c_4xe^{-x}$

Multiplicity of 3?

$c_1e^x + c_2xe^x + c_3x^2e^x$

Solve the given boundary-value problem. (If an answer does not exist, enter DNE.)

11. $y'' - 2y' + 2y = 0, \quad y(0) = 7, \; y(\pi) = 7$

$r^2 - 2r + 2 = 0$

$r^2 - 2r + 1^2 - 1^2 + 2$

$= (r-1)^2 + 1 \overset{SET}{=} 0$

$(r-1)^2 = -1$

$r = 1 \pm i$

$y = e^x(c_1 \cos(x) + c_2 \sin(x))$

$= c_1 e^x \cos(x) + c_2 e^x \sin(x)$

BC: $y(0) = y(\pi) = 7$

$c_1 e^0 \cos(0) + c_2 \sin(0) = 7$

$\boxed{c_1 = 7}$

$c_1 e^{\pi} \cos(\pi) + c_2 e^{\pi} \sin(\pi) = 7$

$c_1 = 7e^{-\pi}$ contradiction.

No Sol'n satisfies both.