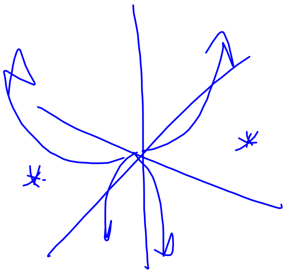
§4.7#30

$f(2,1) = 3 + (2)(1) - 2 - 2$

$= 1$

$f(x,y) = 3 + xy - x - 2y$

probly a saddle



$f(x+\delta, y+\varepsilon)$

$f(x+\delta, y)$

$f(2+\delta, 1)$

$= 3 + (2+\delta)(1) - 2 - \delta - 2$

$= 3 + 2 + \delta - 2 - \delta - 2$

$= 1$

$f(2, 1+\varepsilon) =$

$3 + 2(1+\varepsilon) - 2 - 2(1+\varepsilon)$

$= 3 + 2 + 2\varepsilon - 2 - 2 - 2\varepsilon$

$= 1$

$f(2+\delta, 1+\varepsilon)$

$= 3 + (2+\delta)(1+\varepsilon) - (2+\delta) - 2(1+\varepsilon)$

$= 3 + 2 + 2\varepsilon + \delta + \delta\varepsilon - 2 - \delta - 2 - 2\varepsilon \quad = 1 + \delta\varepsilon$

$\delta, \varepsilon > 0$

$\Rightarrow$ min

$\delta < 0, \varepsilon > 0$

$\Rightarrow$ Not min

§14.2 #37

$$f(x,y)=\begin{cases}\dfrac{x^2y^3}{2x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 1 & \text{if } (x,y)=(0,0)\end{cases}$$

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

Along $x=0$: $\dfrac{0}{y^2}=0 \xrightarrow{y\to 0} 0$

is enough, since @ $(0,0)$, $f(x,y)=1$

$$f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right) & x\neq 0\\ 0 & x=0\end{cases}$$

$\sin\left(\frac{1}{x}\right)$

$x\to$ BIG $\frac{1}{x}\to$ zeroish

is cts everywhere

$x\sin\left(\frac{1}{x}\right)$

$x\in\left(0,\frac{1}{\pi}\right)$

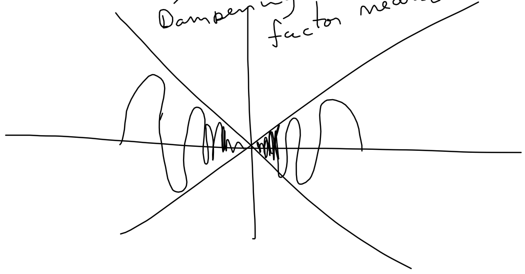
$\Rightarrow \frac{1}{x}\in(\pi,\infty)$

However $x\sin\frac{1}{x}$

$-1\le \sin\left(\frac{1}{x}\right)\le 1$

$x>0$ $\quad -x\le x\sin\left(\frac{1}{x}\right)\le x$

Dampening factor nearer zero.



$$\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)=0$$

$$-x\le x\sin\left(\tfrac{1}{x}\right)\le x$$

$$0\le \lim_{x\to 0} x\sin\left(\tfrac{1}{x}\right)\le 0$$

$$f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right) & x\neq 0\\ 0 & x=0\end{cases}$$ is continuous $\forall x\in\mathbb{R}$

Along $x=c$

$y=c$

$y=x$

$\boxed{y=mx}$