



$f$  is optimized where level curve of  $f$  is parallel to  $g(x, y) = K$ .

$$\nabla f \parallel \nabla g$$

$\nabla f$  points in the direction of GREATEST INCREASE of  $f$ .

$$\nabla f \parallel \nabla g \Rightarrow$$

$$\nabla f = \lambda \nabla g \text{ for some real } \neq \lambda.$$

Method:

$$\text{Find all } x, y, z, \lambda \text{ satisfying } \nabla f = \lambda \nabla g$$

### #s 3 - 17 Use lagrange multipliers to optimize $f$ .

Optimize  $f(x, y) = x^2 + y^2$  Level curves are circles.

Subject to  $g(x, y) = xy = 1$

$$\nabla f = \langle f_x, f_y \rangle = \langle 2x, 2y \rangle$$

$$\nabla g = \langle g_x, g_y \rangle = \langle y, x \rangle$$

$$\nabla f = \lambda \nabla g \Rightarrow$$

$$2x = \lambda y$$

$$2y = \lambda x$$

$$\Rightarrow \lambda = \frac{2x}{y} \Rightarrow 2y = \left(\frac{2x}{y}\right)x = \frac{2x^2}{y}$$

$$\Rightarrow 2y^2 = 2x^2$$

$$\Rightarrow y^2 = x^2$$

$$\Rightarrow y = \pm x$$

But  $xy = 1 \Rightarrow x$  &  $y$  can't be opposite signs

so  $y = x$

so  $xy = 1$  &  $x = y \Rightarrow x = 1 = y$  or  $x = -1 = y$

$$f(1, 1) = 2$$

$$f(-1, -1) = 2$$

Min @ both points

because  $x^2 + y^2$  grows without bound

Maximize  $x+5y = F$  subject to

$$2x+3y \leq 6$$

$$3x-2y \leq 6$$

$$x \geq 0, y \geq 0$$

x	y
0	2
3	0

$0 \leq 6?$   
 $(0,0)$  good

x	y
0	-3
2	0

