§13.2 #s 1-3, 5, 9, 15, 17, 21, 23

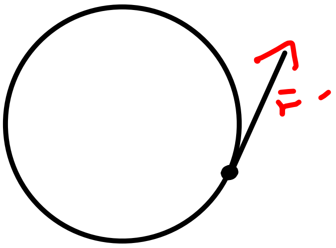
§13.3 #s 1, 4, 7, 10, 13, 15, 17, 21, 28, 47, 50

§13.2 $\bar{r} = \langle f, g, h \rangle$

$\Rightarrow \bar{r}\,' = \langle f', g', h' \rangle$

Unit Tangent Vector $\bar{T} = \frac{1}{\|\bar{r}\,'\|}\bar{r}\,'$

Unit vector in direction of $\bar{r}\,'$



$\boxed{\text{E1}}$

$\bar{r} = \langle 1+t^3, te^{-t}, \sin(2t) \rangle$

$\bar{r}\,' = \langle 3t^2, 1e^{-t} - te^{-t}, 2\cos(2t) \rangle$

Want $\bar{T}\big|_{t=0}$

$$\bar{T} = \frac{1}{\sqrt{(3t^2)^2 + (e^{-t} - te^{-t})^2 + 4\cos^2(2t)}} \boxed{\langle 3t^2, e^{-t} - te^{-t}, 2\cos(2t) \rangle}$$

Now, for $t=0$, we have:

$$\bar{T}(0) = \frac{\langle 0, e^0, 2 \rangle}{\sqrt{0 + (e^0)^2 + 4}} = \frac{1}{\sqrt{5}} \langle 0, 1, 2 \rangle$$

Check $\|\bar{T}\| = \frac{1}{\sqrt{5}}\sqrt{0^2 + 1^2 + 2^2} = \frac{1}{\sqrt{5}}\sqrt{5} = 1$

$$\frac{2}{3} = \frac{1}{3} \cdot 2$$

Props of $\frac{d}{dt}$:

f, g, h diff'l

$\bar{u}, \bar{v}, \bar{r}$ vector funcs

$$\frac{d}{dx}[x^2+5x] = \frac{d}{dx}[x^2] + \frac{d}{dx}[5x] = 2x+5$$

$\boxed{J3}$ ① $(\bar{u} \pm \bar{v})' = \bar{u}' \pm \bar{v}'$

② $(c\bar{v})' = c\bar{v}'$

③ $(f\bar{u})' = f'\bar{u} + f\bar{u}'$

$$(fg)' = f'g + fg'$$

$e^t \langle t, 2, \cos(t) \rangle$

$= \langle te^t, 2e^t, e^t\cos(t) \rangle$ & product rule on the entries.

④ $(\bar{u} \cdot \bar{v})' = \bar{u}' \cdot \bar{v} + \bar{u} \cdot \bar{v}'$

⑤ $(\bar{u} \times \bar{v})' = \bar{u}' \times \bar{v} + \bar{u} \times \bar{v}'$

Chain Rule ⑥ $(\bar{u}(f(t)))' = \left(\frac{du}{df}\right)\left(\frac{df}{dt}\right) = u'(f(t))\, f'(t)$

$\bar{u} = \langle t, \sin t \rangle \Rightarrow \bar{u}(f(t)) = \langle f(t), \sin(f(t)) \rangle$

$f(t) = t^2$

$(\bar{u}(f(t)))' = (\langle 1, \cos(f(t)) \rangle) f'(t)$

$\frac{d}{df}[f]$

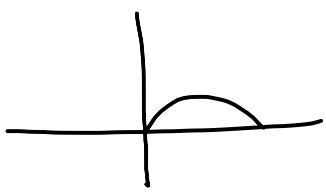
$\boxed{E}$ $(\bar{r} \times \bar{r}')' = \underbrace{\bar{r}' \times \bar{r}'}_{=\bar{0}} + \bar{r} \times \bar{r}'' = \boxed{\bar{r} \times \bar{r}''}$!

$\|\bar{a} \times \bar{b}\| = \|\bar{a}\| \|\bar{b}\| \sin\theta$

$|\bar{a} \cdot \bar{b}| = \|\bar{a}\| \|\bar{b}\| |\cos\theta|$

$\cos\theta$

$\sin\theta \geq 0 \ \forall\ \theta \in [0, \pi]$



Curvature

$$\kappa = \left| \frac{d\bar{T}}{ds} \right|$$ Magnitude of derivative with respect to arc length

So, arc length : 2-D

Recall $s = L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$= \int_{\alpha}^{\beta} \sqrt{(f')^2 + (g')^2}\, dt$$

$x = x(t) = f(t)$

$y = y(t) = g(t)$

3-D

$$s = L = \int_{\alpha}^{\beta} \underbrace{\sqrt{(f')^2 + (g')^2 + (h')^2}\, dt} = \int_{\alpha}^{\beta} |\bar{r}'(t)|\, dt$$

where $\bar{r} = \langle f(t), g(t), h(t) \rangle$

$$ds = \sqrt{(f')^2 + (g')^2 + (h')^2}\, dt$$

$$s = \int_{\alpha}^{\beta} ds$$

Since $|\bar{r}'(t)| \geq 0$

$\Longrightarrow \int_{\alpha}^{t} |\bar{r}'(u)|\, du$

is an increasing function of t.

$$\int_{\alpha}^{5} \sim \; > \int_{\alpha}^{4}$$

FTC I

$f(t)$ $\frac{d}{dt}\int_0^t \sqrt{x^2+5x+\sin x}\,dx$

$= \sqrt{x^2+5x+\sin x}$

$f(3t^2)$ $\frac{d}{dt}\int_0^{3t^2} \sqrt{x^2+5x+\sin x}\,dx$

Chain Rule part

$= \left(\sqrt{(3t^2)^2+5(3t^2)+\sin(3t^2)}\right)(6t)$

$f(w)$

$\frac{d}{dt}\int_0^{w} \sqrt{x^2+5x+\sin x}\,dx = 0$

$\frac{d}{dt}\left[x^2\right] = 0$