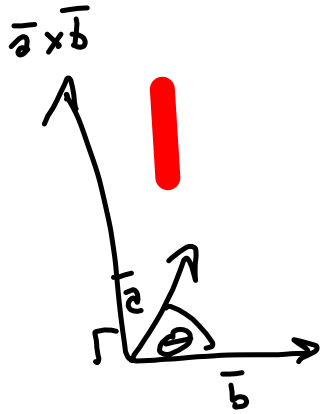
$\bar{a} = \langle 3, 7, -1 \rangle$

$\bar{b} = \langle 1, 5, 6 \rangle$

$\bar{a} \times \bar{b}$



$$\begin{array}{r} \langle 3, 7, -1 \rangle \\ \times \ \langle 1, 5, 6 \rangle \\ \hline \langle 42+5, -(18+1), 15-7 \rangle \end{array}$$

$= \langle 47, -19, 8 \rangle = \bar{c}$

$\bar{a} \cdot \bar{c} = \langle 3, 7, -1 \rangle \cdot \langle 47, -19, 8 \rangle$

$= (3)(47) + (7)(-19) + (-1)(8)$

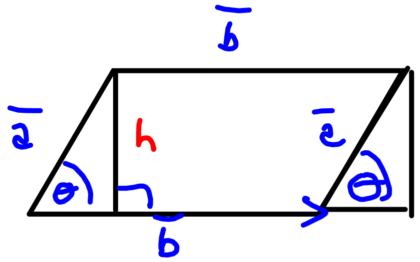
$= 141 - 133 - 8 = 0$

$\|\bar{a} \times \bar{b}\| = \|\bar{a}\| \|\bar{b}\| \sin\theta$

$\|\bar{a} \times \bar{b}\|$ is the

|number|

‖vector‖



$$\frac{h}{\|\bar{a}\|} = \sin\theta$$

$h = \|\bar{a}\| \sin\theta$

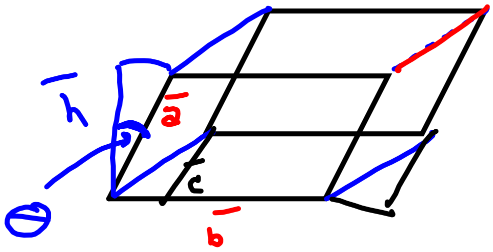
Area $= h \|\bar{b}\|$

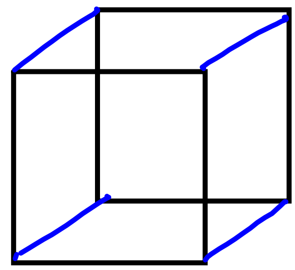
$= \|\bar{a}\| \|\bar{b}\| \sin\theta$

$\|\bar{a} \times \bar{b}\|$ is area of parallelogram defined by $\bar{a}$ & $\bar{b}$

Scalar Triple Product

$|\bar{a}\cdot(\bar{b}\times\bar{c})|$ = Volume of the parallelepiped





$\dfrac{h}{\|\bar{a}\|} = \cos\Theta$

$\|\bar{h}\| = \|\bar{a}\|\cos\Theta$

$\bar{h}$ is vertical $= \langle 0, 0, h\rangle$

$\|\bar{b}\times\bar{c}\|\, h = \|\bar{b}\times\bar{c}\|\,\|\bar{a}\|\cos\Theta$

$= \cancel{\|\bar{b}\times\bar{c}\|}\,\cancel{\|\bar{a}\|}\dfrac{|\bar{a}\cdot(\bar{b}\times\bar{c})|}{\cancel{\|\bar{a}\|}\,\cancel{\|\bar{b}\times\bar{c}\|}}$ = volume.

$\cos\Theta = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}$ where Θ is angle between $\bar{u}$ & $\bar{v}$.

$\bar{a} = \langle 3, 7, -1 \rangle$

$\bar{b} = \langle 1, 5, 6 \rangle$

$\bar{a} \times \bar{b}$

$$\begin{array}{r} \langle 3, 7, -1 \rangle \\ \times \ \langle 1, 5, 6 \rangle \\ \hline \langle 42 + 5, -(18+1), 15 - 7 \rangle \end{array}$$

$= \langle 47, -19, 8 \rangle = \bar{c}$

$$\begin{array}{l} \langle 3, 7, -1 \rangle, 3, 7 \\ \langle 1, 5, 6 \rangle, 1, 5 \\ \hline \langle 42 + 5, -1 - 18, 15 - 7 \rangle \end{array}$$

$\langle 47, -19, 8 \rangle$

$\bar{a} \cdot (\bar{b} \times \bar{c})$

$\langle 1, 2, 3 \rangle = \bar{a}, \ \langle -2, 3, 5 \rangle = \bar{b}, \ \langle 2, -7, 1 \rangle = \bar{c}$

$\bar{a} \cdot (\bar{b} \times \bar{c}) = \langle 1, 2, 3 \rangle \cdot \langle 38, 12, 8 \rangle$

$= 38 + 24 + 24 = \boxed{86}$

$\bar{b} = \langle -2, 3, 5 \rangle, -2, 3$

$\bar{c} = \langle 2, -7, 1 \rangle, 2, -7$

$\bar{b} \times \bar{c} = \langle 38, 12, 8 \rangle$

$$\begin{array}{r} \langle -2, 3, 5 \rangle \\ \times\ \langle 2, -7, 1 \rangle \\ \hline \langle 38, -(-12), 8 \rangle \end{array}$$

$\bar{i} \times \bar{j} = \bar{k}$ $\qquad \bar{j} \times \bar{i} = -\bar{k}$

$\bar{j} \times \bar{k} = \bar{i}$ $\qquad \bar{k} \times \bar{j} = -\bar{i}$

$\bar{k} \times \bar{i} = \bar{j}$ $\qquad \bar{i} \times \bar{k} = -\bar{j}$

$(5\bar{a}) \times \bar{b} = 5(\bar{a} \times \bar{b}) = \bar{a} \times (5\bar{b})$

respects scalar multiples.

$(\bar{a} + \bar{b}) \times \bar{c} = \bar{a} \times \bar{c} + \bar{b} \times \bar{c}$

5. $\bar{a} \cdot (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \cdot c$

6. wha? Put it on cheat sheet.

§12.