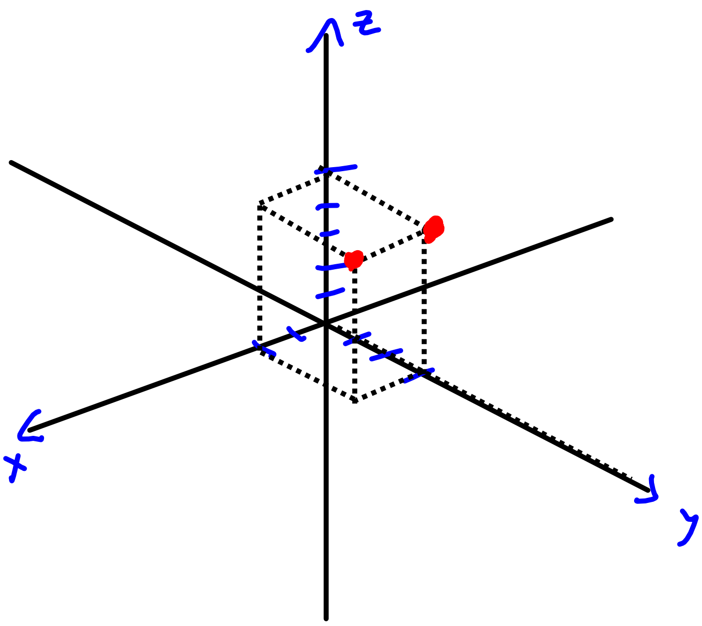
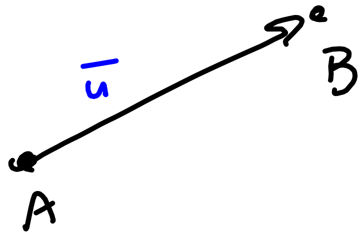
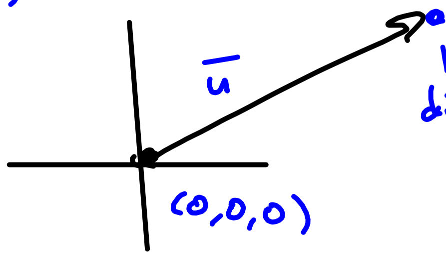

z
x
y

Vector: Directed Line Segment



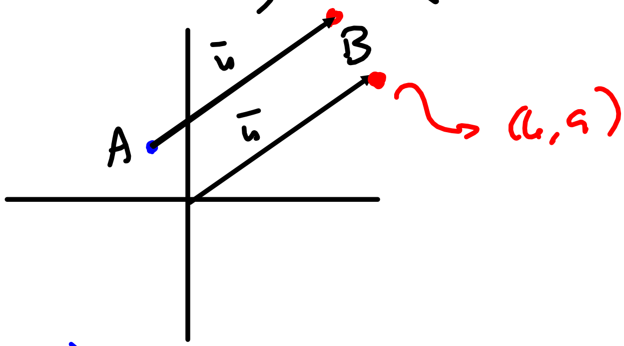
$\overrightarrow{AB}$

Vector in standard position has same direction and length as $\overrightarrow{AB} = \bar{u}$, but its butt abuts the origin.



$\|\bar{u}\|$ in direction of $\overrightarrow{AB} = \bar{u}$.

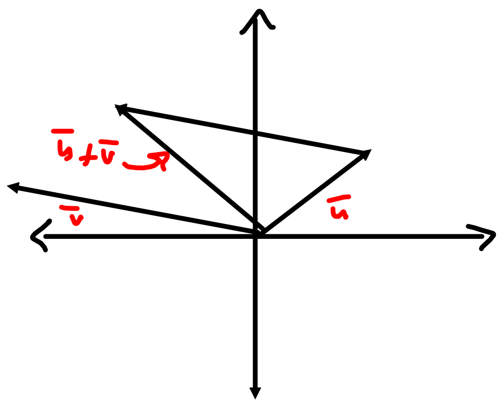
$A(-1,2)$, $B=(5,11)$



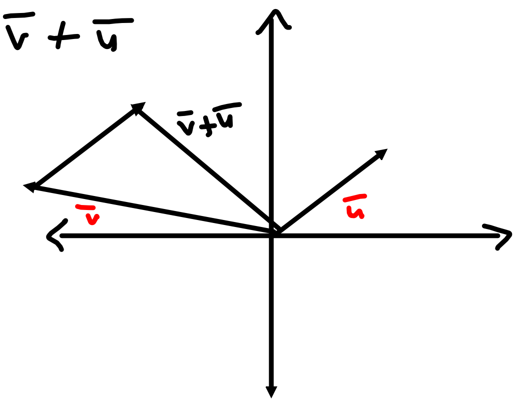
$\bar{u} = \langle 5-(-1), 11-2 \rangle = \langle 6,9 \rangle$

Adding vectors. $\bar{u}+\bar{v}$

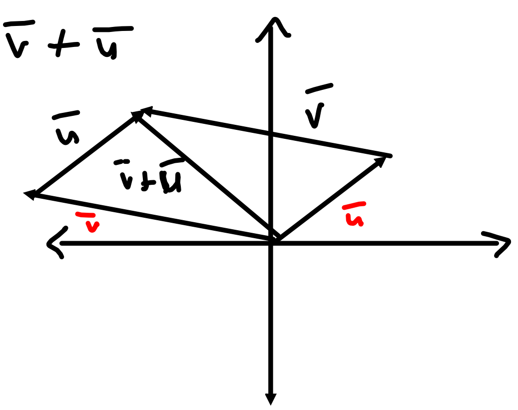
$\bar{u} = \langle 2,2 \rangle$
$\bar{v} = \langle -5,1 \rangle$



$\bar{v}+\bar{u}$

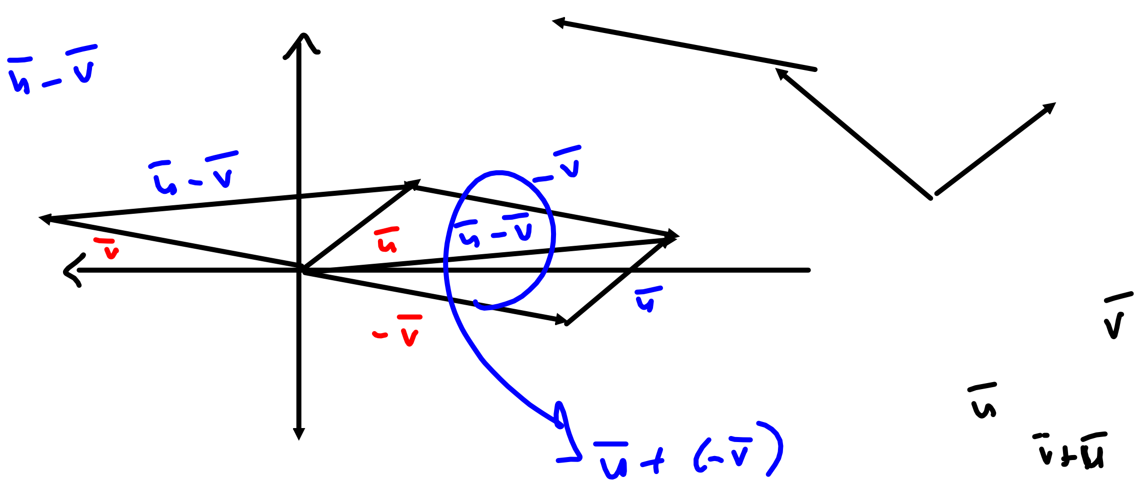


$\bar{v}+\bar{u}$



$\bar{u}-\bar{v}$ is the vector that starts at the tip of $\bar{v}$ and ends at the tip of $\bar{u}$.

$(\bar{u}+(-\bar{v}))$



$\bar{u} = \langle u_1, u_2 \rangle$

$\|\bar{u}\| = \sqrt{u_1^2 + u_2^2}$

3-D:

$\bar{u} = \langle u_1, u_2, u_3 \rangle$

$\|\bar{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$

$\bar{u} + \bar{v} = \bar{v} + \bar{u}$ Addition of vectors is commutative.

$\bar{u} = \langle u_1, u_2 \rangle$, $\bar{v} = \langle v_1, v_2 \rangle$

Then $\bar{u} + \bar{v} = \langle u_1 + v_1, u_2 + v_2 \rangle = \langle v_1 + u_2, v_2 + u_2 \rangle$

$= \bar{v} + \bar{u}$

Also associative:

$(\bar{u} + \bar{v}) + \bar{w} = \bar{u} + (\bar{v} + \bar{w})$

$(a+b)+c$
$= a + (b+c)$

$\exists \ \bar{0} \ni \bar{u} + \bar{0} = \bar{u} \ \forall \bar{u} \in \mathbb{R}^3$

There is / There exist — such that / so that — for all, for each, for every

The existence of an additive identity.

$\bar{0} = \langle 0, 0, 0 \rangle$

Special! Canonical (standard) Basis Vectors,

$\bar{i} = \langle 1, 0, 0 \rangle$

$\bar{j} = \langle 0, 1, 0 \rangle$

$\bar{k} = \langle 0, 0, 1 \rangle$

$\bar{u} = \langle 1, 2, 3 \rangle = 1\bar{i} + 2\bar{j} + 3\bar{k}$

$= 1 \langle 1, 0, 0 \rangle + 2 \langle 0, 1, 0 \rangle + 3 \langle 0, 0, 1 \rangle$

This is called a linear combo on
$\bar{i}, \bar{j}, \bar{k}$ $\quad a\bar{i} + b\bar{j} + c\bar{k}$, where $a, b, c \in \mathbb{R}$

Out of sequence:
Scalar multiplication
Let $a \in \mathbb{R}$ and $\bar{u} \in \mathbb{R}^3$
then $a\bar{u} = a\langle u_1, u_2, u_3\rangle = \langle au_1, au_2, au_3\rangle$

$\bar{v} = 3\langle 1, 2, 3\rangle = \langle 3, 6, 9\rangle$

Give a unit vector in the direction of $\bar{v}$.

$\underbrace{\text{unit vector}}_{\text{Length} = 1}$

$\bar{v} = \langle 3, 6, 9\rangle$

Hint: any $c\bar{v}$ is in the direction of $\bar{v}$.

$\|\bar{v}\| = \sqrt{3^2+6^2+9^2} = \sqrt{9+36+81} = \sqrt{126}$

$= 3\sqrt{21}$

Consider $\bar{w} = \frac{1}{\|\bar{v}\|}\bar{v}$

$$\begin{array}{r|l} 2 & 126 \\ 3 & 63 \\ 3 & 21 \\ & 7 \end{array}$$

$= \frac{1}{3\sqrt{21}}\langle 3,6,9\rangle = \langle \frac{3}{3\sqrt{21}}, \frac{6}{3\sqrt{21}}, \frac{9}{3\sqrt{21}}\rangle$

what's its length?

$\sqrt{\left(\frac{3}{3\sqrt{21}}\right)^2+\left(\frac{6}{3\sqrt{21}}\right)^2+\left(\frac{9}{3\sqrt{21}}\right)^2}$

$= \sqrt{\left(\frac{1}{3\sqrt{21}}\right)^2(3^2+6^2+9^2)}$

$\frac{1}{3\sqrt{21}}\sqrt{9+36+81} = \frac{1}{3\sqrt{21}}\sqrt{126}$

$= \frac{1}{3\sqrt{21}}\cdot 3\sqrt{21} = 1$

$\S$12.2 #s 2-4, 6, 8-10, 15, 16, 19, 20, 23, 24, 26-33, 36, 41, 43, 46