1. Evaluate the line integral over the given curve C.

$\int_C 4xy\,ds$, where C is the line segment joining (–4, –5) to (5, 4)

1. ANS:
$234\sqrt{2}$

PTS: 1 DIF: Medium REF: 16.2.4
NOT: Section 16.2

$\bar{r} = (1-t)\langle -4,-5\rangle + t\langle 5,4\rangle$

$= \langle -4,-5\rangle + \langle 4t,5t\rangle + \langle 5t,4t\rangle$

$= \langle -4,-5\rangle + \langle 9t,9t\rangle$

$= \langle 9t-4, 9t-5\rangle$

$ds = \sqrt{x_t^2+y_t^2}\,dt = \sqrt{9^2+9^2}\,dt$

$= \sqrt{2}\sqrt{81}\,dt = 9\sqrt{2}\,dt$

$4\int_C xy\,ds = \int_0^1 (9t-4)(9t-5)\,9\sqrt{2}\,dt$

$= 4 \cdot 9\sqrt{2}\int_0^1 (81t^2 - 81t + 20)\,dt$

$36\sqrt{2}\left[27t^3 - \frac{81}{2}t^2 + 20t\right]_0^1$

$= 36\sqrt{2}\left[27 - \frac{81}{2} + 20\right] = 36\sqrt{2}\left[\frac{94-81}{2}\right]$

$= 18\sqrt{2}\,(13) = 234\sqrt{2}$

81
64
145

$145 \div 5 = 29$

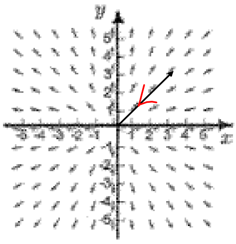
$-36t$
$-45t$
$-81t$

$47 \times 2 = 94$

$24 + 180 = 204$

18×13: $54 + 180 = 234$

2. The plot of a vector field is shown below. A particle is moved from the point $(3,3)$ to $(0,0)$. By inspection, determine whether the work done by **F** on the particle is positive, negative, or zero.



2. ANS:
negative

PTS: 1 DIF: Medium REF: 16.2.17a
NOT: Section 16.2

3. Evaluate the line integral over the given curve C.

   $\int_C 4xy\,ds$, where C is the line segment joining $(-2, -1)$ to $(4, 5)$

3. ANS:
   $120\sqrt{2}$

   PTS: 1 DIF: Medium REF: 16.2.2
   NOT: Section 16.2

4. Determine whether **F** is conservative. If so, find a function f such that $\mathbf{F} = \nabla f$..

$\mathbf{F}(x,y,z) = 9x^2y^4z^2\mathbf{i} + 12x^3y^3z^2\mathbf{j} + 6x^3y^4z\mathbf{k}$

4. ANS:
$f(x,y,z) = 3x^3y^4z^2 + C$

PTS: 1 DIF: Easy REF: 16.3.3
NOT: Section 16.3

continuous partials everywhere

$\Rightarrow$ Conservative if curl $\bar{F} = \bar{0}$

$$\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \times \left\langle 9x^2y^4z^2, 12x^3y^3z^2, 6x^3y^4z \right\rangle$$

$$\text{curl}\,\bar{F} = \nabla \times F = \langle 24x^3y^3z - 24x^3y^3z, (18x^2y^4 - 18x^2y^4), 36x^2y^3z^2 - 36x^2y^3z^2 \rangle$$

$$= \bar{0}. \Rightarrow \text{yes, conservative}$$

Finding potential function, f :

$$f_x = 9x^2y^4z^2 \Rightarrow f = 3x^3y^4z^2 + \alpha(y,z)$$

$$\Rightarrow f_y = 12x^3y^3z^2 + \alpha_y(y,z) = 12x^3y^3z^2$$

$$\Rightarrow \alpha_y(y,z) = 0$$

$$\Rightarrow \alpha(y,z) \equiv \alpha(z)$$

$$\Rightarrow f = 3x^3y^4z^2 + \alpha(z)$$

$$\Rightarrow f_z = 6x^3y^4z + \alpha'(z) = 6x^3y^4z + 0 \Rightarrow$$

$$\alpha'(z) = 0$$

$$\therefore f(x,y,z) = 3x^3y^4z^2 + C$$

5. Determine whether $\mathbf{F}$ is conservative. If so, find a function f such that $\mathbf{F} = \nabla f$..

$$\mathbf{F}\left(x,y,z\right) = (6\sinh 2z)\mathbf{i} + \left(3e^{5x}\cos 3y\right)\mathbf{j} + (12x\cosh 2z)\mathbf{k}$$

5. ANS:

The vector field $\mathbf{F}\left(x,y,z\right) = (6\sinh 2z)\mathbf{i} + \left(3e^{5x}\cos 3y\right)\mathbf{j} + (12x\cosh 2z)\mathbf{k}$ is not conservative. There exists no scalar field f such that $\mathbf{F} = \nabla f$.

PTS: 1 DIF: Easy REF: 16.3.10
NOT: Section 16.3

6. Let R be a plane region of area A bounded by a piecewise-smooth simple closed curve C. Using Green's Theorem, it can be shown that the centroid of R is $(\bar{x}, \bar{y})$, where

$$\bar{x} = \frac{1}{2A}\oint_C x^2\,dy \qquad \bar{y} = -\frac{1}{2A}\oint_C y^2\,dx$$

Use these results to find the centroid of the given region.

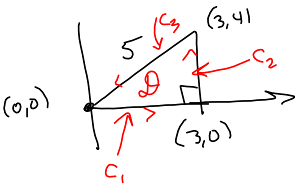
The triangle with vertices $(0,0)$, $(3,0)$, and $(3,4)$.

6. ANS:
$\bar{x} = 2;\ \bar{y} = \frac{4}{3}$

PTS: 1 DIF: Medium REF: 16.4.23
NOT: Section 16.4

$\boxed{\text{Better question:}}$

Calculate $\int_C x^2 dy$ in 2 ways



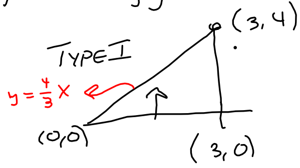
$A = \frac{1}{2}bh = \frac{1}{2}(3)(4) = 6$

$\therefore \frac{1}{2A} = \frac{1}{2(6)} = \boxed{\frac{1}{12} = \frac{1}{2A}}$

Green's Version: $\int_C x^2 dy = \int_C 0dx + x^2 dy = \iint_D (Q_x - P_y)\,dA = \iint 2x\,dA$

$P = 0,\ Q = x^2$

$P_y = 0,\ Q_x = 2x$



Using No Green's

$$\int_C x^2 dy = \sum_{k=1}^{3} \int_{C_k} x^2 dy$$

C_1: $\int_{C_1} x^2 \cdot 0 = 0$

C_2: $\int_{C_2} = \int_0^4 3^2 dy = [9y]_0^4 = 36$

$x=3$ is fixed!

C_3: $(1-t)\langle 3,4\rangle + t\langle 0,0\rangle$

$= \langle 3-3t, 4-4t\rangle$

$(3(1-t))^2$
$= 9(t^2 - 2t + 1)$
$= 9t^2 - 18t + 9$

$\int_{C_3} x^2 dy = \int_0^1 (9t^2 - 18t + 9)\cdot(-4dt)$

$y = 4 - 4t$

$\Rightarrow dy = -4dt$

This gives

$-4\left[3t^3 - 9t^2 + 9t\right]_0^1$

$= -4[3 - 9 + 9]$

$= -4[3] = \boxed{-12}$

C_3

$\sum \int_{C_k} x^2 dy = -12 + 36$
$= 24$

$\frac{1}{2A}\int x^2 dy = \frac{24}{12} = \boxed{2}$

Using Green's:

$m = \frac{4-0}{3-0} = \frac{4}{3}$

$y = m(x - x_0) + y_0$

$y = \frac{4}{3}x$

$$\int_0^3 \int_0^{\frac{4}{3}x} 2x\,dy\,dx$$

$$= \int_0^3 [2xy]_0^{\frac{4}{3}x} dx$$

$$= \int_0^3 \left[\frac{8}{3}x^2\right] dx$$

$$= \frac{8}{9}x^3\Big]_0^3$$

$$= \frac{8}{9}(27) = \boxed{24}$$

$\therefore \bar{x} = \frac{1}{2A}\int_C x^2 dy = \frac{1}{12}\cdot 24$

$= 2.$

7. Find (a) the divergence and (b) the curl of the vector field **F**.

$\mathbf{F}(x,y,z) = \cos z\mathbf{i} + 5y\sin 3z\mathbf{j} + 4x^2 z\mathbf{k}$

$\bar{F} = \langle P, Q, R \rangle$

a $\operatorname{div}\bar{F} = \nabla \cdot \bar{F} = P_x + Q_y + R_z$

$= 0 + \boxed{5\sin(3z) + 4x^2}$

b $\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$

$\times \; \langle \cos(z), 5y\sin(3z), 4x^2 z \rangle$

$\langle 0 - 15y\cos(3z), -(8xz - (-\sin z)), 0 - 0 \rangle$

$\nabla \times \bar{F} = \langle -15y\cos(3z), \sin z - 8xz, 0 \rangle$

7. ANS:
(a). $4x^2 + 5\sin 3z$
(b). $-15y\cos 3z\mathbf{i} - (8xz + \sin z)\mathbf{j}$

PTS: 1 DIF: Medium REF: 16.5.4
NOT: Section 16.5

8. Let f be a scalar field. Determine whether the expression is meaningful. If so, state whether the expression represents a scalar field or a vector field.

curl f

8. ANS:
The curl is a property of vector fields, not scalar fields. So, curl f is not meaningful.

PTS: 1 DIF: Medium REF: 16.5.12a MSC: Short Answer
NOT: Section 16.5

9. Let **F** be a vector field. Determine whether the expression is meaningful. If so, state whether the expression represents a scalar field or a vector field.

$\nabla \cdot (\nabla \times \mathbf{F})$

9. ANS:
$\nabla \times \mathbf{F}$ is the curl of **F**, so it is a vector field. Thus, $\nabla \cdot (\nabla \times \mathbf{F})$ is the divergence of a vector field, which is a scalar field. Assuming all the partial derivatives are defined and continuous, $\nabla \cdot (\nabla \times \mathbf{F})$ is meaningful.

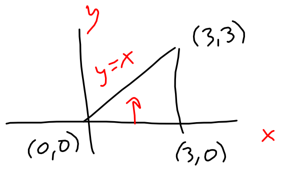
PTS: 1 DIF: Medium REF: 16.5.121 MSC: Short Answer
NOT: Section 16.5

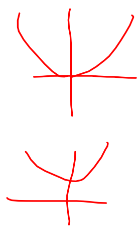
10. Find the area of the surface S where S is the part of the plane $z = 2x^2 + y$ that lies above the triangular region with vertices (0, 0), (3, 0), and (3, 3).

10. ANS:
$$\frac{73\sqrt{146} - \sqrt{2}}{24}$$

$$\frac{1}{24}\sqrt{2}\,(73\sqrt{73} - 1)$$

PTS: 1 DIF: Medium REF: 16.6.44
NOT: Section 16.6





$y=0$

$y=1$

$$A(S) = \int_0^3 \int_0^x dS$$

$S: z = 2x^2 + y$, so dS is found...

$$\text{Area} = \int$$

$\langle x, y, 2x^2+y \rangle$

This should be a magnitude

$$dS = (\bar{r}_x \times \bar{r}_y)\,dA$$

$$\langle 1, 0, 4x \rangle$$
$$\times \langle 0, 1, 1 \rangle$$
$$\langle -4x, -1, 1 \rangle$$

I eventually got it right.

$$\Rightarrow \|\bar{r}_x \times \bar{r}_y\| = \sqrt{16x^2 + 1 + 1} = \sqrt{16x^2+2}$$

$$= \int_0^3 \int_0^x \sqrt{16x^2+2}\,dy\,dx$$

11. Find the area of the surface S where S is the part of the sphere $x^2+y^2+z^2=16$ that lies to the right of the xz-plane and inside the cylinder $x^2+z^2=9$.

y

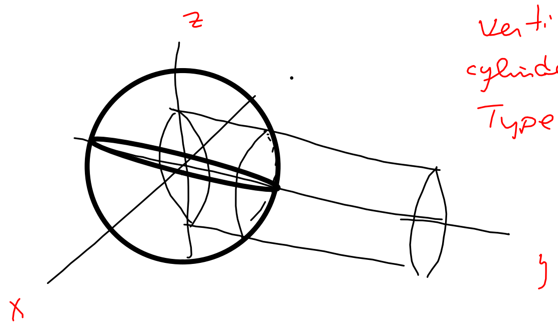
$x^2+y^2=9$

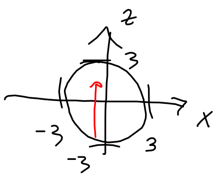
$z=\pm\sqrt{16-x^2-y^2}$ if this were right-side up with $z=f(x,y)$ as the cap

11. ANS:
$16\pi\left(8-\sqrt{55}\right)$

PTS: 1 DIF: Difficult REF: 16.6.45
NOT: Section 16.6

Test: vertical cylinder. Type I solid





$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\sqrt{\frac{x^2+z^2}{16-x^2-z^2}+1}\,dz\,dx$$

$$\vec{r}=\left\langle x,\sqrt{16-x^2-z^2},z\right\rangle$$

$$\int_0^{2\pi}\int_0^3\sqrt{\frac{r^2}{16-r^2}+1}\,r\,dr\,d\theta$$

$$\vec{r}_x \quad \left\langle 1,\frac{-x}{\sqrt{16-x^2-z^2}},0\right\rangle$$

$$\vec{r}_z \quad \left\langle 0,\frac{-z}{\sqrt{16-x^2-z^2}},1\right\rangle$$

$$\left\langle -\frac{x}{\sqrt{16-x^2-z^2}},-(+1),\frac{-z}{\sqrt{16-x^2-z^2}}\right\rangle$$

$$\iint_D dS=\iint_D\|\vec{r}_x\times\vec{r}_z\|\,dA$$

$$\|\vec{r}_x\times\vec{r}_z\|=\sqrt{\frac{x^2+z^2}{16-x^2-z^2}+1}$$

$$=\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\sqrt{\frac{x^2+z^2}{16-x^2-z^2}+1}\,dz\,dx$$

$$\cancel{=\sqrt{\frac{16}{16-x^2-z^2}}=\frac{4}{\sqrt{16-x^2-z^2}}}$$

FAILING TO GET provided solution.

Solution to the similar problem referenced in the text 16.6.45:

39–50 Find the area of the surface.

45. The part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$

45. $z = f(x, y) = xy$ with $x^2 + y^2 \le 1$, so $f_x = y$, $f_y = x \quad \Rightarrow$

$$A(S) = \iint_D \sqrt{1 + y^2 + x^2}\, dA = \int_0^{2\pi} \int_0^1 \sqrt{r^2 + 1}\, r\, dr\, d\theta = \int_0^{2\pi} \left[\tfrac{1}{3} \left(r^2 + 1\right)^{3/2} \right]_{r=0}^{r=1} d\theta$$

$$= \int_0^{2\pi} \tfrac{1}{3}\left(2\sqrt{2} - 1\right) d\theta = \tfrac{2\pi}{3}\left(2\sqrt{2} - 1\right)$$

Notice, how, in practice, we side-step writing the iterated integral, in rectangular coordinates. Makes sense. I always want to try to write the thing, both ways, and evaluate both integrals on a machine, as a double-check.

1

12. Find the area of the surface S where S is the part of the sphere $x^2+y^2+z^2=1$ that lies inside the cylinder $x^2-x+y^2=0$.

12. ANS:
$2(\pi-2)$

PTS: 1 DIF: Difficult REF: 16.6.50
NOT: Section 16.6

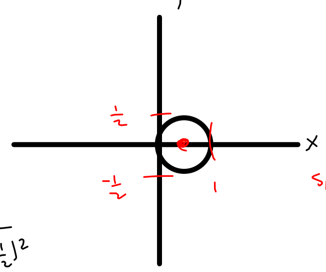
$\left(x^2-x+\left(\frac{1}{4}\right)\right)-\frac{1}{4}+y^2$

$\left(x-\frac{1}{2}\right)^2+y^2=\frac{1}{4}$

$(h,k)=\left(\frac{1}{2},0\right)$

$r=\frac{1}{2}$

$y=\sqrt{\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}$



Double the area of one. This sphere intersects the cylinder in 2 places.

$$2\int_0^1\int_{-\sqrt{\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}}^{\sqrt{\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}}$$

$x^2+y^2+z^2=1$

$x^2+y^2+z^2=1$

$\left(x-\frac{1}{2}\right)^2+y^2=\frac{1}{4}$

$x^2-\left(x-\frac{1}{2}\right)^2+z^2=\frac{3}{4}$

$y=\sqrt{1-x^2-z^2}$

$y_x=\dfrac{-x}{\sqrt{1-x^2-z^2}}$, $y_z=\dfrac{-z}{\sqrt{1-x^2-z^2}}$

$$2\int_0^1\int_{-\sqrt{\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}}^{\sqrt{\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}}\left(\frac{x^2+z^2}{1-x^2-z^2}\right)^{\frac{1}{2}}\,dz\,dx$$

$\bar{r}_x=\left\langle 1,-\dfrac{x}{\sqrt{1-x^2-z^2}},0\right\rangle$

$\times\quad \bar{r}_z=\left\langle 0,\dfrac{-z}{\sqrt{1-x^2-z^2}},1\right\rangle$

$\left\langle\dfrac{-x}{\sqrt{1-x^2-z^2}},-1,\dfrac{-z}{\sqrt{1-x^2-z^2}}\right\rangle$

$\|\bar{r}_y\times\bar{r}_y\|=\sqrt{\dfrac{x^2+z^2}{1-x^2-z^2}+1}$

$\left(x-\frac{1}{2}\right)^2+y^2=\frac{1}{4}$

$x^2-x+y^2=\frac{1}{4}$

$r^2\cos^2x-r\cos x+r^2\sin^2x=\frac{1}{4}$

Solution to the similar problem referenced in the text 16.6.50:

50. The cylinder encloses separate portions of the sphere in the upper and lower halves. The top half of the sphere is $z = f(x, y) = \sqrt{b^2 - x^2 - y^2}$ and D is given by $\{(x, y) \mid x^2 + y^2 \le a^2\}$. By Formula 9, the surface area of the upper enclosed portion is

$$A = \iint_D \sqrt{1 + \left(\frac{-x}{\sqrt{b^2 - x^2 - y^2}}\right)^2 + \left(\frac{-y}{\sqrt{b^2 - x^2 - y^2}}\right)^2}\, dA = \iint_D \sqrt{1 + \frac{x^2 + y^2}{b^2 - x^2 - y^2}}\, dA$$

$$= \iint_D \sqrt{\frac{b^2}{b^2 - x^2 - y^2}}\, dA = \int_0^{2\pi} \int_0^a \frac{b}{\sqrt{b^2 - r^2}}\, r\, dr\, d\theta = b \int_0^{2\pi} d\theta \int_0^a \frac{r}{\sqrt{b^2 - r^2}}\, dr$$

$$= b\left[\theta\right]_0^{2\pi} \left[-\sqrt{b^2 - r^2}\right]_0^a = 2\pi b\left(-\sqrt{b^2 - a^2} + \sqrt{b^2 - 0}\right) = 2\pi b\left(b - \sqrt{b^2 - a^2}\right)$$

The lower portion of the sphere enclosed by the cylinder has identical shape, so the total area is $2A = 4\pi b\left(b - \sqrt{b^2 - a^2}\right)$.

13. Find an equation in rectangular coordinates, and then identify the surface.

$r(u,v) = 6v\mathbf{i} + (8u - v)\mathbf{j} + (u + 6v)\mathbf{k}$

$\bar{r} = \langle 6v,\ 8u-v,\ u+6v \rangle$

13. ANS:
Answers may vary.
$49x + 6y - 48z = 0$; plane

PTS: 1 DIF: Easy REF: 16.6.3

$\bar{r}_u : \langle 0, 8, 1 \rangle$

$\times\ \bar{r}_v : \langle 6, -1, 6 \rangle$

$\langle 49, 6, -48 \rangle = \bar{n}$

So $\bar{n} \cdot (\bar{x} - \bar{x}_0) = \bar{0}$

If $(x_0, y_0, z_0) \in \mathcal{P}$, then if (x,y,z) is also, we have

$\bar{x} = \langle x-x_0, y-y_0, z-z_0 \rangle$ is a vector in the plane.

$(x_0, y_0, z_0) = (0,0,0)$

$\bar{n} \cdot \bar{x} = 0$

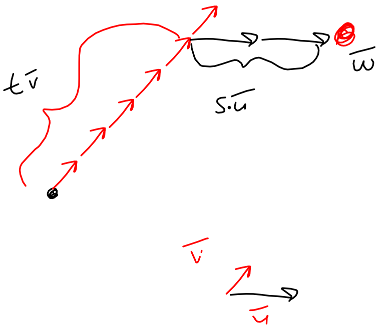
$49(x-x_0) + 6(y-y_0) - 48(z-z_0) = 0$

$\longrightarrow$

$49x + 6y - 48z = 0.$

14. Find a vector representation for the surface.

The plane that passes through the point $(2, 5, 1)$ and contains the vectors $2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ $= \bar{u}$ and $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$.. $= \bar{v}$



14. ANS:
Answers may vary.
$\mathbf{r}(u, v) = (2 + 2u + 2v)\mathbf{i} + (5 + 5u - 3v)\mathbf{j} + (1 - 3u + 5v)\mathbf{k}$

PTS: 1 DIF: Medium REF: 16.6.19

$$\bar{r}(s,t) = \bar{x}_0 + s\bar{u} + t\bar{v}$$
$$= \langle 2,5,1\rangle + s\langle 2,5,-3\rangle + t\langle 2,-3,5\rangle$$
for $s, t \in \mathbb{R}$.

15. Find an equation of the tangent plane to the parametric surface represented by r at the specified point.

$\mathbf{r}(u,v) = (u^2 - v^2)\mathbf{i} + u\mathbf{j} + v\mathbf{k}$; $(0,3,3)$ $\to \bar{x}_0$

$\bar{r}_u = \langle 2u, 1, 0 \rangle$

$\times \quad \bar{r}_v = \langle -2v, 0, 1 \rangle$

$\langle 1, -2u, 2v \rangle = \bar{n}$

$\bar{n} \cdot (\bar{x} - \bar{x}_0) = 0$

$\langle 1, -2u, 2v \rangle \cdot \langle x-0, y-3, z-3 \rangle$

$= \quad x - 2u(y-3) + 2v(z-3) = 0$

$= x - 6(y-3) + 6(z-3) = 0$

Need (u,v) for

$\bar{x}_0 = \langle 0, 3, 3 \rangle$

$0 = u^2 - v^2$

$3 = u$

$3 = v$

15. ANS:
Answers may vary.
$x - 6y + 6z = 0$

PTS: 1 DIF: Medium REF: 16.6.38

16. Find an equation of the tangent plane to the parametric surface represented by **r** at the specified point.

$\mathbf{r}(u,v) = ue^v\mathbf{i} + uv\mathbf{j} + ve^{-u}\mathbf{k}$; $u = \ln 5$, $v = 0$

16. ANS:
Answers may vary.
$y - 5\ln 5z = 0$

PTS: 1 DIF: Medium REF: 16.6.35
NOT: Section 16.6

$\bar{r}(\ln 5, 0)$

$= \langle (\ln 5)e^0, 0, 0\rangle$

$(\ln 5, 0, 0) = P_0$

$\langle \ln 5, 0, 0\rangle = \bar{x}_0$

$\bar{x} = \langle x, y, z\rangle$

$\bar{x} - \bar{x}_0 = \langle x - \ln 5, y, z\rangle$

$\bar{r}_u = \langle e^v, v, -ve^{-u}\rangle$

$e^{-\ln 5} = \frac{1}{e^{\ln 5}} = \frac{1}{5}$

$\bar{r}_v = \langle ue^v, u, e^{-u}\rangle$

Plug in $u = \ln 5$, $v = 0$:

$\bar{r}_u(\ln 5, 0) = \langle 1, 0, 0\rangle$

$\times$ $\bar{r}_v(\ln 5, 0) = \langle \ln 5, \ln 5, \frac{1}{5}\rangle$

$\langle 0, -(\frac{1}{5}), \ln 5\rangle = \bar{n}$

$\bar{n} \cdot (\bar{x} - \bar{x}_0) = 0$:

$\langle 0, -\frac{1}{5}, \ln 5\rangle \cdot \langle x - \ln 5, y, z\rangle$

$= 0$ $\boxed{-\frac{1}{5}y + \ln 5\, z = 0}$

OR

$-y + 5\ln z = 0$

$\boxed{y - 5\ln z = 0}$

17. Find an equation of the tangent plane to the parametric surface represented by **r** at the specified point.

$\mathbf{r}(u,v) = ue^{v}\mathbf{i} + uv\mathbf{j} + ve^{-u}\mathbf{k}$; $u = \ln 9$, $v = 0$

17. ANS:
Answers may vary.
$y - 9\ln 9z = 0$

PTS: 1 DIF: Medium REF: 16.6.36

18. Use the Divergence Theorem to find the flux of **F** across S; that is, calculate $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$.

$\mathbf{F}(x,y,z) = (9xy + \cos z)\mathbf{i} + (x - \sin z)\mathbf{j} + 4xz\mathbf{k}$; S is the sphere $x^2 + y^2 + z^2 = 4$

18. ANS:
0

PTS: 1 DIF: Difficult REF: 16.6.9

$$\iint_S \bar{F}\cdot\bar{n}\,dS = \iiint_E \operatorname{div}\bar{F}\,dV$$

$S: \rho = 4$

$$\operatorname{div}\bar{F} = 9y + 0 + 4x = 4x + 9y$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{2} (9y + 4x)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{2} (9\cos\phi\sin\theta + 4\cos\phi\cos\theta)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= 0$$

$$\iint_D \bar{F}\cdot\bar{n}\,dS$$

$\rho = \text{constant} = 2$

$\bar{n}: \quad \bar{r} = \langle 2\cos\phi\cos\theta,\ 2\cos\phi\sin\theta,\ 2\sin\phi\rangle$

Homework
1st few questions
in 16.9:
Set up to both ways.
(Don't evaluate.)

$\bar{r}_\phi \quad \langle -2\sin\phi\cos\theta,\ -2\sin\phi\sin\theta,\ \cancel{2\cos\phi}\rangle$

$\bar{r}_\theta \quad \langle -2\cos\phi\sin\theta,\ 2\cos\phi\cos\theta,\ \cancel{0}\rangle$

$\langle -4\cos^2\phi\cos\theta,\ -(4\cos^2\phi\sin\theta),\ -4\cos^2\phi\sin\theta\cos\theta - 4\cos\phi\sin\phi\sin^2\theta\rangle$

ExamView references 16.6.9, and here it is, and as expected, has nothing to do with Divergence Theorem, in Section 16.9.

7–12 Use a computer to graph the parametric surface. Get a printout and indicate on it which grid curves have u constant and which have v constant.

9. $\mathbf{r}(u, v) = \langle u \cos v, u \sin v, u^5 \rangle$,
$-1 \leqslant u \leqslant 1,\ 0 \leqslant v \leqslant 2\pi$

Here's 16.9.9, from the text. It's the "fat sphere" that Scott asked about.

5–15 Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$; that is, calculate the flux of $\mathbf{F}$ across S.

9. $\mathbf{F}(x, y, z) = x^2 \sin y\, \mathbf{i} + x \cos y\, \mathbf{j} - xz \sin y\, \mathbf{k}$,
S is the "fat sphere" $x^8 + y^8 + z^8 = 8$

And here's the solution to that one:

9. $\operatorname{div} \mathbf{F} = 2x \sin y - x \sin y - x \sin y = 0$, so by the Divergence Theorem, $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E 0\, dV = 0$.

And this is a great test question. But I'd like to throw in one that's not trivial, too.

19. Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl}\mathbf{F} \cdot d\mathbf{S}$.

$\mathbf{F}(x,y,z) = 4xy\mathbf{i} + 5yz\mathbf{j} + 2z^2\mathbf{k}$;

19. ANS:
0

PTS: 1 DIF: Medium REF: 16.8.3

S is the part of the ellipsoid $9x^2 + 9y^2 + 4z^2 = 36$ lying above the xy-plane and oriented with normal pointing upward.

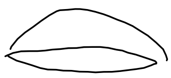
$\bar{F} = \langle 4xy, 5yz, 2z^2 \rangle$

$\nabla \times \bar{F}: \quad \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$

$\times \quad \langle 4xy, 5yz, 2z^2 \rangle$

$\langle 0 - 5y, -(0-0), 0 - 4x \rangle$

$= \langle -5y, 0, -4x \rangle$ Not zero, shucks!



above xy-plane.
Boundary curve C is the circle:

$C: \quad 9x^2 + 9y^2 = 36, \quad z = 0$

$9(x^2+y^2) = 9r^2 = 36$

$r^2 = 4$

$r = \pm 2 \rightarrow r = 2$

$0 \le t \le 2\pi$

$\int_0^{2\pi} \bar{F} \cdot d\bar{r}$

$= \int_0^{2\pi} \bar{F}(\bar{r}(t)) \cdot \bar{r}\,'(t)\, dt$

$\bar{F}(x,y,z) = \langle 4xy, 5yz, 2z^2 \rangle$

$\bar{r}(t) = \langle x(t), y(t), z(t) \rangle$

$= \langle 2\cos t, 2\sin t, 0 \rangle$

$\bar{r}\,'(t) = \langle -2\sin t, 2\cos t, 0 \rangle$

LOL!

$\bar{F}(\bar{r}(t)) = \langle 4(2\cos t)2\sin t, 5(\sin t)(0), 0 \rangle$

$\bar{F} \cdot \bar{r}\,' = -32\cos t \sin^2 t$

$$\int_0^{2\pi} \bar{F} \cdot d\bar{r} = -32\int_0^{2\pi} (\sin^2 t)\cos(t)\, dt = -32\left[\frac{\sin^3(t)}{3}\right]_0^{2\pi} = 0$$

20. Use Stokes' Theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

$\mathbf{F}(x,y,z) = 7z\mathbf{i} + y\mathbf{j} + 4xz\mathbf{k}$;

C is the boundary of the triangle with vertices $(6,0,0)$, $(0,6,0)$, and $(0,0,6)$ oriented in a counterclockwise direction when viewed from above

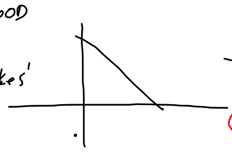
20. ANS: -18

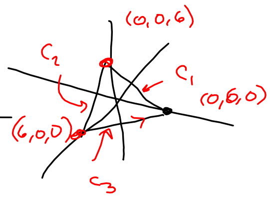
PTS: 1 DIF: Medium REF: 16.8.4

This one's a $\iint$, where #19 went the other way.

$\vec{F} = \langle 7z, y, 4xz \rangle$

Ugh. Good thing for Stokes'





C_1: $\vec{r}$: $(1-t)\langle 0,0,6\rangle$

$+ t\langle 6,0,0\rangle$

$= \langle 0,0,6-t\rangle + \langle 6t,0,0\rangle = \langle 6t, 0, 6-t\rangle$

$\int_{C_1} = \int_0^1$

Looks pretty painful!

So, I GUESS we can use Stokes' Theorem!

Here's the solution to the referenced problem. Not at all what we want, here. We want Stokes going the other way, like #

4. The boundary curve C is the circle $y^2 + z^2 = 4$, $x = 2$ which should be oriented in the counterclockwise direction when viewed from the front, so a vector equation of C is $\mathbf{r}(t) = 2\,\mathbf{i} + 2\cos t\,\mathbf{j} + 2\sin t\,\mathbf{k}$, $0 \le t \le 2\pi$. Then $\mathbf{F}(\mathbf{r}(t)) = \tan^{-1}(32\cos t\sin^2 t)\,\mathbf{i} + 8\cos t\,\mathbf{j} + 16\sin^2 t\,\mathbf{k}$, $\mathbf{r}'(t) = -2\sin t\,\mathbf{j} + 2\cos t\,\mathbf{k}$, and $\mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) = -16\sin t\cos t + 32\sin^2 t\cos t$. Thus

$$\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = \oint_C \mathbf{F}\cdot d\mathbf{r} = \int_0^{2\pi} \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,dt = \int_0^{2\pi}(-16\sin t\cos t + 32\sin^2 t\cos t)\,dt$$

$$= \left[-8\sin^2 t + \tfrac{32}{3}\sin^3 t\right]_0^{2\pi} = 0$$