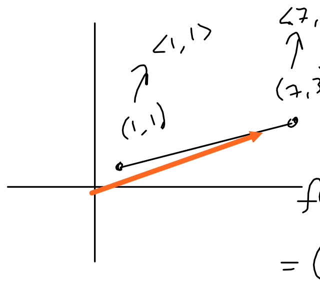
MAT 203 Test 4 – spring, 2019 Name________________________

100 Points Covers Chapter 16 NO GRAPHING CALCULATORS!!!

Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes!

Setups count far more than final answers. I hope that you don't need a calculator for this test.

1. (10 pts) Evaluate the line integral $\int_C xy^2 ds$, where C is the line segment from (1,1) to (7,3). Writing the definite integral is 8 pts.



$(1-t)\langle 1,1\rangle + t\langle 7,3\rangle$

$= \langle 1-t+7t, 1-t+3t\rangle$

$= \bar{r}(t) = \langle 6t+1, 2t+1\rangle$ $0 \le t \le 1$

$\bar{r}'(t) = \langle 6,2\rangle$

$ds = \|\bar{r}'(t)\|\,dt$

$\|\bar{r}'(t)\| = \sqrt{6^2+2^2}$

$= \sqrt{40} = 2\sqrt{10}$

$f(x,y) = xy^2 = (6t+1)(2t+1)^2$

$= (6t+1)(4t^2+4t+1)$

$= 24t^3 + 24t^2 + 6t$

$\quad\quad 4t^2 + 4t + 1$

$24t^3 + 28t^2 + 10t + 1$

$\int_0^1 (24t^3+28t^2+10t+1)\,2\sqrt{10}\,dt$

$= 2\sqrt{10}\left[6t^4 + \frac{28}{3}t^3 + 5t^2 + t\right]_0^1$

$= 2\sqrt{10}\left[\frac{6t^4}{1}\cdot\frac{3}{3} + \frac{28t}{3} + \frac{5t^3}{1}\cdot\frac{3}{3} + \frac{t}{1}\cdot\frac{3}{3}\right]_0^1$

$= \frac{2\sqrt{10}}{3}\left[18 + 28 + 15 + 3\right]$

$46 + 18 = 64$

$= \frac{128\sqrt{10}}{3} = \int_C xy^2 ds$

2. (10 pts) (2-D) Let $\bar{F} = \langle \cos(y), -x\sin(y) \rangle$. Find the function f such that $\nabla f = \bar{F}$. (Remember this "variation of parameters" idea for Differential Equations:)

$\exists f \ni \nabla f = \bar{F} \Rightarrow \bar{F} = \langle \cos(y), -x\sin(y) \rangle = \langle f_x, f_y \rangle$

"There exists an f such that the gradient of f is $\bar{F}$."

$f_x = \cos(y) \Rightarrow f = x\cos y + g(y)$

$f_y = -x\sin(y) \Rightarrow x\cos y + h(x) \Rightarrow h(x) = g(y) = C \overset{SET}{\equiv} 0$

$\Rightarrow$ $f(x) = x\cos(y)$

3. (10 pts) (3-D) Let $\bar{F} = \langle \cos(y), -x\sin(y), 0 \rangle$. Use *curl* $\bar{F}$ to show that $\bar{F}$ is a conservative vector field.

I could never remember $Q_x - P_y$, but I could learn the curl & embed all 2-vectors in $\mathbb{R}^3$ & do Green's by learning Stokes!

$$\text{curl}\,\bar{F} = \nabla \times \bar{F} : \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}$$

$$\times \quad \langle \cos(y), -x\sin(y), 0 \rangle, \cos(y), -x\sin(y)$$

$$\langle 0, 0, -\sin(y) + \sin(y) \rangle = \bar{0}$$

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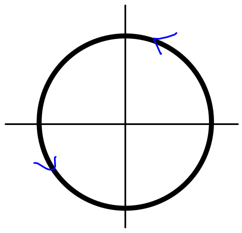
4. (10 pts) Let C be the circle $x^2 + y^2 = 4$. Evaluate the line integral $\int_C (xy+2)\,ds$. The farther you can take your setup, the more points this is worth. The final answer is worth 2 points.



$r = 2$

$\vec{r} = \langle 2\cos\theta, 2\sin\theta \rangle$

$\vec{r}\,' = \langle -2\sin\theta, 2\cos\theta \rangle$

$\|\vec{r}\,'(\theta)\| = \sqrt{4\sin^2\theta + 4\cos^2\theta}$

$= \sqrt{4(\sin^2\theta + \cos^2\theta)}$

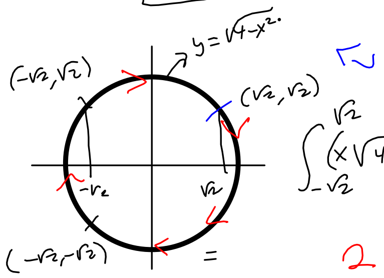
$= \sqrt{4} = 2$

So $ds = \boxed{2\,d\theta}$

$$\int_C (xy+2)\,ds = \int_0^{2\pi} (2\cos\theta\, 2\sin\theta + 2)(2\,d\theta)$$

$$= 8\int_0^{2\pi} \sin\theta\cos\theta\, d\theta + 4\int_0^{2\pi} d\theta$$

$$= \frac{8}{2}\sin^2\theta \Big]_0^{2\pi} + 4\theta\Big]_0^{2\pi} = 8\pi$$



$$\int_{-\sqrt{2}}^{\sqrt{2}} \left(x\sqrt{4-x^2} + 2\right) ds$$

$ds = \|\vec{r}\,'(x)\| = \dfrac{2}{\sqrt{4-x^2}}$

$\vec{r}(x) = \langle x, \sqrt{4-x^2} \rangle$

$\vec{r}\,'(x) = \langle 1, \frac{1}{2}(4-x^2)^{-\frac{1}{2}}(-2x) \rangle$

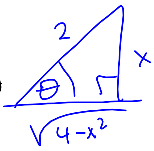
$= \langle 1, \dfrac{-x}{\sqrt{4-x^2}} \rangle$

$\Rightarrow \|\vec{r}\,'(x)\| = \sqrt{1^2 + \dfrac{x^2}{4-x^2}}$

$= \dfrac{\sqrt{4-x^2+x^2}}{\sqrt{4-x^2}} = \dfrac{2}{\sqrt{4-x^2}}$

$$\int_C (xy+2)\,ds = \int_{-\sqrt{2}}^{\sqrt{2}} \left(x\sqrt{4-x^2}\right)\left(\frac{2}{\sqrt{4-x^2}}\right)dx + \int_{-\sqrt{2}}^{\sqrt{2}} \frac{4}{\sqrt{4-x^2}}\,dx$$

$$= 2\int_{-\sqrt{2}}^{\sqrt{2}} x\,dx + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{4}{\sqrt{4-4\sin^2\theta}}\, 2\cos\theta\, d\theta$$



$\frac{x}{2} = \sin\theta$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

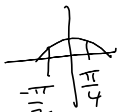
$$= 0 + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{4}{2\cos\theta}\, 2\cos\theta\, d\theta = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 4\,d\theta = 4\left(\frac{\pi}{4}\right)(2)$$

$= 2\pi$

$-\frac{\pi}{4}$ to $\frac{\pi}{4}$:

$2\sin\theta = -\sqrt{2}$

$2\sin\theta = \sqrt{2}$



The other 3 line integrals in rectangular coordinates are all the same.

5. (10 pts) Evaluate the line integral $\int_C \overline{F} \bullet d\overline{r}$, where $\overline{F} = \langle 2x, 3y \rangle$ and C is the circle $x^2 + y^2 = 4$ with positive orientation, and likewise viewing your $\overline{r}$ as a circle living in the xy-plane as $\overline{r} = \langle 2\cos(\theta), 2\sin(\theta), 0 \rangle$.
The final answer is 1 point.

6. (10 pts) Evaluate the integral in #5 using Stokes' Theorem, by viewing $\overline{F} = \langle 2x, 3y, 0 \rangle$ as a vector living in $\mathbf{R}^3$ (3-space, you know, space). You may also use the (vector form of) Green's Theorem for this.

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7. (10 pts) Find the surface area of the portion of the plane $z = 2x + 4$ that lies within the cylinder $x^2 + y^2 = 4$. Expressing the integral is the main thing. The farther you can take it, the better, but keep in mind the actual value is worth only 1 point.

8. (10 pts) Find the flux of the vector field $\mathbf{F} = \langle 3x, 2y, -x \rangle$ through the portion of the plane $z = 2x + 4$ that lies inside the cylinder $x^2 + y^2 = 4$. That is, find $\iint_S \bar{F} \bullet d\bar{S}$. Do this as a surface integral, directly.

9. (10 pts) Use the Divergence Theorem to evaluate $\iint_S \overline{F} \bullet d\overline{S}$ from #8.

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10. (10 pts) Find an equation in rectangular coordinates for the tangent plane to the surface $\bar{r}(r,\theta)=\langle 3r\cos(\theta), 2r\sin(\theta), 5r\rangle$, $r\geq 0, 0\leq\theta\leq 2\pi$ at the point $(3,0,5)$.

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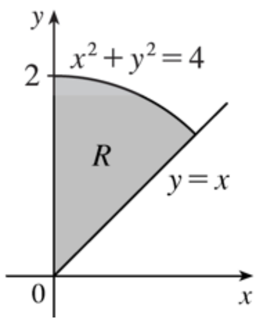
Bonus Section

1. (Bonus) (10 pts) Use the picture to evaluate the iterated integral $\iint_R x^2 y\, dA$ by converting to POLAR coordinates.



2. (Bonus) (10 pts) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the equation

$xyz - x\cos\left(x^2 y\right) = z^2$.

3. (Bonus) (10 pts) Find the first partial f_x for $f(x,y)=\int_{\cos(x^2)}^{y^3}\frac{t^4\tan(t^2)}{t^2+5}\,dt$.

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4. (Bonus) (10 pts) Express the plane in #10 as a vector equation.

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