11–14 Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. (Check the orientation of the curve before applying the theorem.)

12. $\mathbf{F}(x, y) = \langle y^2 \cos x, x^2 + 2y \sin x \rangle$, $= \langle P, Q \rangle \Longrightarrow$ $Q_x = 2x + 2y \cos x$

C is the triangle from $(0, 0)$ to $(2, 6)$ to $(2, 0)$ to $(0, 0)$ $\quad P_y = 2y \cos x$

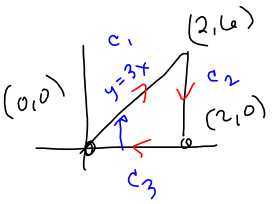
$$\int_C \bar{F} \cdot dr = \int_0^2 \int_0^{3x} (Q_x - P_y)\, dy\, dx$$

$$\int_C \bar{F} \cdot d\bar{r} = \int_C P dx + Q dy = \iint_{\mathcal{D}} (Q_x - P_y)\, dA$$

$Q_x = 2x + 2y \cos x \Longrightarrow Q_x - P_y = 2x$

$P_y = 2y \cos x$

$m = \frac{6-0}{2-0} = \frac{6}{2} = 3 \Longrightarrow y = 3(x-0) + 0 = 3x$



$$= \int_0^2 \int_0^{3x} (2x + 2y \cos x - 2y \cos x)\, dy\, dx$$

$$= \int_0^2 \int_0^{3x} 2x\, dy\, dx = \int_0^2 \Big[2xy\Big]_0^{3x} dx$$

$$= \int_0^2 2x(3x)\, dx = 6 \int_0^2 x^2\, dx = 6 \cdot \frac{x^3}{3}\Big]_0^2$$

$$= 2 \cdot 2^3 = 16$$

$d\bar{r} = \bar{r}'(t)\,dt$

C_1: $(0,0)$ to $(2,6)$

$(1-t)\langle 0\ 0\rangle + t\langle 2,6\rangle = \langle 2t, 6t\rangle = \bar{r}_1 \Rightarrow \bar{r}_1' = \langle 2,6\rangle$

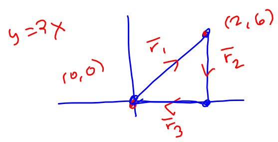
C_2: $(2,6)$ to $(2,0)$

$(1-t)\langle 2,6\rangle + t\langle 2,0\rangle = \langle 2-2t+2t, 6-6t+0\rangle$

$= \langle 2, 6-6t\rangle = \bar{r}_2 \rightarrow \bar{r}_2' = \langle 0,-6\rangle$

C_3: $(2,0)$ to $(0,0)$

$(1-t)\langle 2,0\rangle + t\langle 0,0\rangle = \langle 2-2t, 0\rangle = \bar{r}_3 \Rightarrow \bar{r}_3' = \langle -2,0\rangle$



Our orientation is clockwise!

$$= \int_0^1 \bar{F}\cdot\langle 2,6\rangle\,dt + \int_0^1 \bar{F}\cdot\langle 0,-6\rangle\,dt + \int_0^1 \bar{F}\cdot\langle 2,0\rangle\,dt$$

$\bar{F}(x,y) = \langle y^2\cos x,\ x^2+2y\sin x\rangle$ for all 3 line segments

$\langle 2t, 6t\rangle = \bar{r}_1 \Rightarrow \bar{r}_1' = \langle 2,6\rangle$

$$\int_C \bar{F}\cdot d\bar{r} = \sum_{k=1}^{3}\int_{C_k}\bar{F}\cdot d\bar{r} \underset{C_1}{=} \int_0^1 \langle (6t)^2\cos(2t),\ (2t)^2+2(6t)\sin(2t)\rangle\cdot\langle 2,6\rangle\,dt$$

$\langle 2, 6-6t\rangle = \bar{r}_2 \rightarrow \bar{r}_2' = \langle 0,-6\rangle$

$$+ {}_{C_2}\int_0^1 \langle (6-6t)^2\cos(2),\ 2^2+2(6-6t)\sin(2)\rangle\cdot\langle 0,-6\rangle\,dt$$

$\langle 2-2t, 0\rangle = \bar{r}_3 \Rightarrow \bar{r}_3' = \langle -2,0\rangle$

$$+ {}_{C_3}\int_0^1 \langle 0^2\cos(2-2t),\ (2-2t)^2+2(0)\sin(2-2t)\rangle\cdot\langle 2,0\rangle\,dt$$

$\bar{F}(x,y) = \langle y^2\cos x,\ x^2+2y\sin x\rangle$

$$= \int_0^1 2(36t^2)\cos(2t) + 6\left(4t^2+12t\sin(2t)\right)dt$$

$$+ \int_0^1 \left(0 - 6\left(4+2(6-6t)\sin(2)\right)\right)dt$$

$$+ \int_0^1 0\,dt = \int_0^1 \left(72t^2\cos(2t) + 24t^2 + 72t\sin(2t) - 24 - 12(6-6t)\sin(2)\right)dt$$

$$= \int_0^1 \left(72t^2\cos(2t) + 24t^2 + 72t\sin(2t) - 24 - 72\sin(2) + 72t\sin(2)\right)dt$$