

Section 15.3 Double/Iterated Integrals over General Regions

S 15.3 #s 1, 8, 15, 19, 20, 31, 48, 53 from Handout.

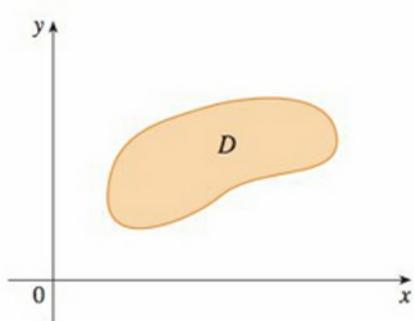


FIGURE 1

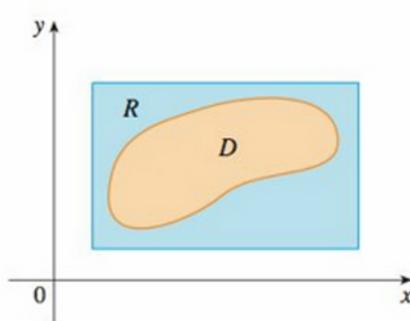
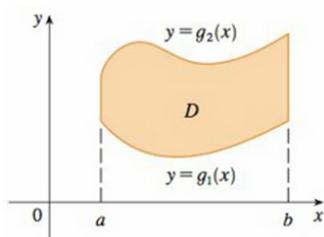


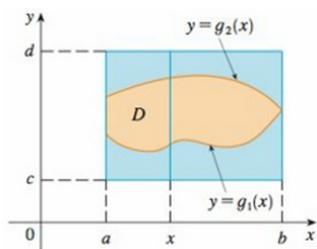
FIGURE 2

$$F(x, y) = \begin{cases} f(x, y) & \text{if } (x, y) \text{ is in } D \\ 0 & \text{if } (x, y) \text{ is in } R \text{ but not in } D \end{cases}$$



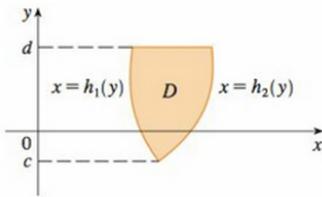
TYPE I

$$\iint_D f(x, y) dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) dy dx$$



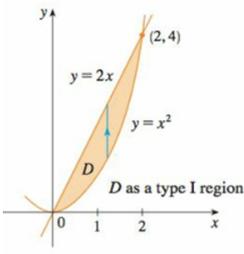
0 4

TYPE II

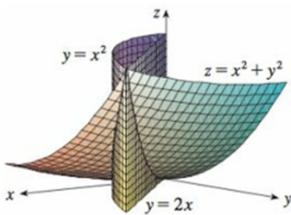


$$\iint_D f(x, y) dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) dx dy$$

EXAMPLE 2 Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in the xy -plane bounded by the line $y = 2x$ and the parabola $y = x^2$.



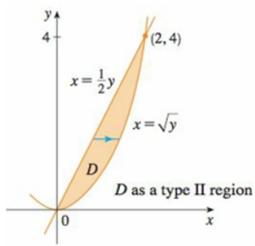
$$\begin{aligned} \text{TI } V &= \int_0^2 \int_{x^2}^{2x} (x^2 + y^2) dy dx \\ &= \int_0^2 \left[x^2 y + \frac{1}{3} y^3 \right]_{x^2}^{2x} dx \\ &= \int_0^2 \left(x^2 [2x] + \frac{1}{3} [2x]^3 - \left(x^2 [x^2] + \frac{1}{3} [x^2]^3 \right) \right) dx \\ &= \int_0^2 \left(2x^3 + \frac{8}{3} x^3 - \left(x^4 + \frac{1}{3} x^6 \right) \right) dx \\ &= \int_0^2 \left[\frac{14}{3} x^3 - x^4 - \frac{1}{3} x^6 \right] dx = \left(\frac{14}{3} \cdot \frac{1}{4} x^4 - \frac{1}{5} x^5 - \frac{1}{21} x^7 \right) \Big|_0^2 \\ &= \frac{7}{6} (2^4) - \frac{1}{5} (2^5) - \frac{1}{21} (2^7) - [0] = \\ &= \frac{7 \cdot 16}{3} - \frac{32}{5} - \frac{128}{21} = \frac{56}{3} - \frac{32}{5} - \frac{128}{21} = \\ &= \frac{56(105) - 32(63) - 128(15)}{3 \cdot 5 \cdot 21} \end{aligned}$$



$$\begin{array}{r} 5080 \\ 1900 \\ \hline 6980 \\ - 1016 \\ \hline 5964 \end{array}$$

$$\begin{array}{r} 63 \\ 32 \\ \hline 1126 \\ 1890 \\ \hline 1016 \\ \hline 1280 \\ 640 \\ \hline 1920 \end{array}$$

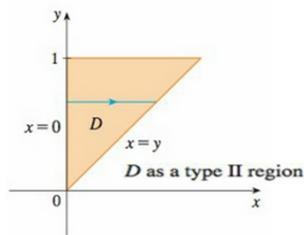
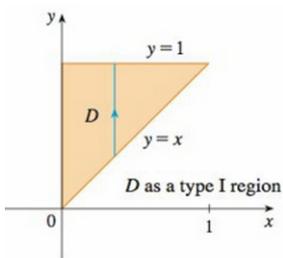
Nope!



TYPE II ∴

$$\int_0^4 \int_{x=\frac{1}{2}y}^{x=\sqrt{y}} (x^2 + y^2) dx dy = \frac{216}{35}, \text{ by Maple.}$$

EXAMPLE 5 Evaluate the iterated integral $\int_0^1 \int_x^1 \sin(y^2) dy dx$. is TYPE I formulation



$\int_0^1 \int_0^y \sin(y^2) dx dy$ TYPE II formulation makes it Ely

$$= \int_0^1 \sin(y^2) \int_0^y dx dy$$

$$= \int_0^1 \sin(y^2) [x]_0^y dy$$

$$= \int_0^1 \sin(y^2) [y - 0] dy$$

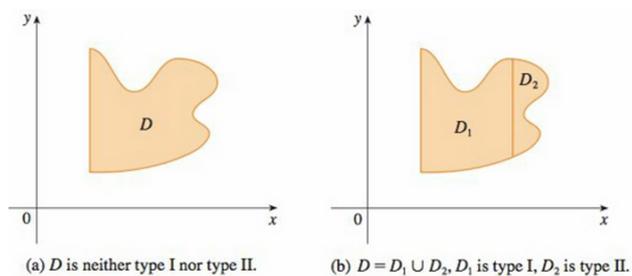
$$= \frac{1}{2} \int_0^1 2y \sin(y^2) dy = \frac{1}{2} \int_0^1 \sin(u) du$$

$$= \frac{1}{2} [-\cos(y^2)]_0^1 = -\frac{1}{2} [\cos(1) - \cos(0)]$$

$$= -\frac{1}{2} [\cos(1) - 1]$$

$$\boxed{\frac{1}{2} [1 - \cos(1)]}$$

We can partition the domain D in order to make things work.



$$\iint_D 1 \, dA = A(D)$$

If $m \leq f(x, y) \leq M$ for all (x, y) in D , then $mA(D) \leq \iint_D f(x, y) \, dA \leq MA(D)$

