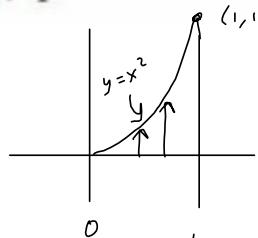


7-18 Evaluate the double integral.

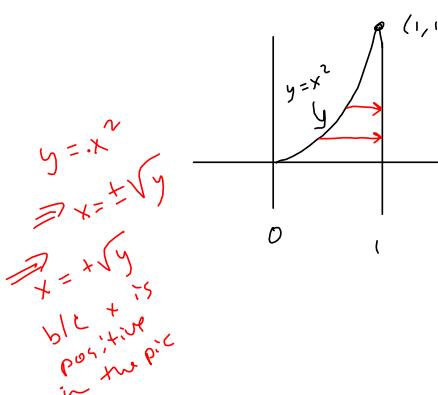
8. $\iint_D \frac{y}{x^5 + 1} dA, D = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq x^2\}$



$$\begin{aligned} & \text{I} \quad \int_0^1 \int_0^{x^2} \frac{y}{x^5 + 1} dy dx \quad \text{FUBINI!} \\ &= \int_0^1 \frac{1}{x^5 + 1} \left(\frac{y^2}{2} \right)_0^{x^2} dx \quad \text{Don't think so!} \\ &= \int_0^1 \frac{1}{x^5 + 1} \left[\frac{x^4}{2} \right] dx = \frac{1}{2} \int_0^1 \frac{x^4}{x^5 + 1} dx = \frac{1}{2} \cdot \frac{1}{5} \int_0^1 \frac{5x^4}{x^5 + 1} dx = \frac{1}{10} \left[\ln|x^5 + 1| \right]_0^1 \end{aligned}$$

$$u = x^5 + 1 \Rightarrow du = 5x^4 dx \quad \int u^{-1} du = \ln|u| + C$$

$$= \frac{1}{10} [\ln(2) - \ln(1)] = \boxed{\frac{\ln(2)}{10}}$$



$$\text{II} \quad \int_0^1 \int_{\sqrt{y}}^1 \left(\frac{y}{x^5 + 1} \right) dx dy \quad \text{is how it's}$$

written, but only a cruel math god would require us to evaluate this, this way.

$$= \frac{40 \ln(2)}{(5 + \sqrt{5})^2 (-5 + \sqrt{5})^2}, \text{ obviously.}$$

$$\begin{aligned} & (5 + \sqrt{5})^2 (-5 + \sqrt{5})^2 \\ &= (5 + \sqrt{5})^2 (5 - \sqrt{5})^2 = ((5 + \sqrt{5})(5 - \sqrt{5}))^2 \\ &= (25 - 5)^2 = 20^2 = 400, \text{ so} \end{aligned}$$

$$\frac{40 \ln(2)}{(5 + \sqrt{5})^2 (-5 + \sqrt{5})^2} = \frac{40 \ln(2)}{400} = \frac{\ln(2)}{10}.$$

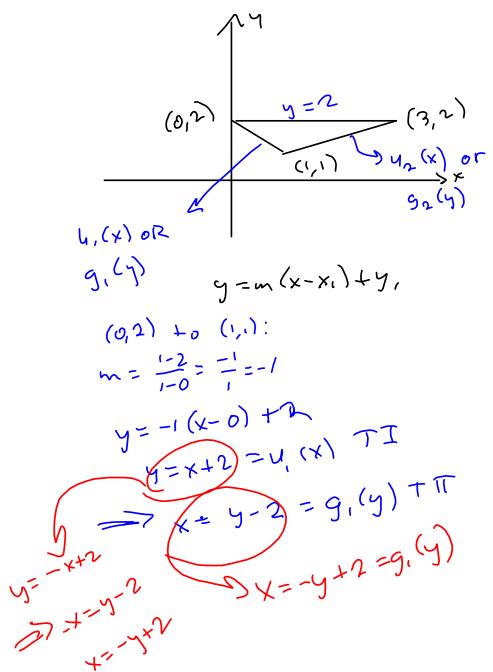
Sweet!

0.6931471804e-1

$\approx .069314718$

$$15. \iint_D y^3 dA,$$

D is the triangular region with vertices $(0, 2)$, $(1, 1)$, $(3, 2)$



This can be viewed as 2 T I integrals

$$\int_0^1 \int_{u_1(x)}^{2} y^3 dy dx + \int_1^3 \int_{u_2(x)}^{2} y^3 dy dx$$

or

as a single T II integral (preferred)

$$\int_1^2 \int_{g_1(y)}^{g_2(y)} y^3 dx dy$$

$$\text{T I: } \int_0^1 \int_{x+1}^{2} y^3 dy dx + \int_1^3 \int_{\frac{1}{2}x+1}^{2} y^3 dy dx$$

$(1, 1) \rightarrow (3, 2)$

$$m = \frac{2-1}{3-1} = \frac{1}{2}$$
 $y = \frac{1}{2}(x-1) + 1$
 $y = \frac{1}{2}x + \frac{1}{2} = u_2(x)$

T I

$$y - \frac{1}{2} = \frac{1}{2}x$$
 $x = 2y - 1 = g_2(y)$
 $u = \frac{1}{2}x + \frac{1}{2}$
 $du = \frac{1}{2}dx$

$$\text{T II} = \left[4x - \frac{(x+2)^4}{20} \right]_0^1 + \left[4x - 2 \left[\frac{(\frac{1}{2}x+\frac{1}{2})^4}{20} \right] \right]_1^3$$

Sloppy, cramped.

BAD!

$$\begin{aligned} &= \left[4 - \frac{243}{20} \right] - \left[0 - \frac{3^4}{20} \right] \\ &+ \left[12 - 2 \left[\frac{3^4}{20} \right] \right] - \left[4 - 2 \left[\frac{1}{20} \right] \right] \\ &= \frac{80-243}{20} + \frac{32}{20} + \frac{240-64}{20} - \left[\frac{80-2}{20} \right] \\ &= \frac{80-243+32+240-64-78}{20} \\ &= \frac{352-385}{20} = \text{Negative?} \end{aligned}$$

OMG!

19–28 Find the volume of the given solid.

19. Under the plane $x + 2y - z = 0$ and above the region bounded by $y = x$ and $y = x^4$

45-50 Evaluate the integral by reversing the order of integration.

48. $\int_0^1 \int_x^1 e^{x/y} dy dx$

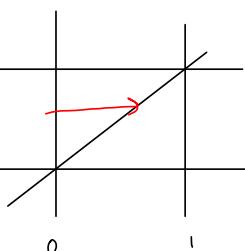
$$y=x \Rightarrow$$

$$x=y \text{ Duh.}$$

$$\text{So, } \int_0^1 \int_0^y e^{x/y} dx dy$$

$$u = \frac{x}{y} = \frac{1}{y} \cdot x \text{ and } \frac{1}{y} \text{ is held constant, so}$$

$$du = \frac{1}{y} dx$$



$$\int_0^1 \int_0^y y e^{\frac{x}{y}} \cdot \frac{1}{y} dx dy = \int_0^1 \left[y e^{\frac{x}{y}} \right]_0^y dy = \int_0^1 [ye^1 - ye^0] dy$$

$$= \int_0^1 (e-1)y dy = (e-1) \left[\frac{y^2}{2} \right]_0^1 = \frac{e-1}{2}$$

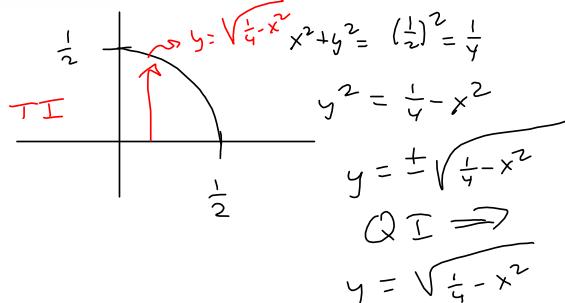
II If $m \leq f(x, y) \leq M$ for all (x, y) in D , then

$$mA(D) \leq \iint_D f(x, y) dA \leq MA(D)$$

53-54 Use Property 11 to estimate the value of the integral.

53. $\iint_Q e^{-(x^2+y^2)} dA$, Q is the quarter-circle with center the origin and radius $\frac{1}{2}$ in the first quadrant

$$e^{-\frac{(x^2+y^2)^2}{4}} = \frac{1}{e^{\frac{(x^2+y^2)^2}{4}}} \quad \text{T I:} \\ \int_0^{\frac{1}{2}} \int_0^{\sqrt{\frac{1}{4}-x^2}} e^{-\frac{(x^2+y^2)^2}{4}} dy dx$$



$\frac{1}{e^{\frac{(x^2+y^2)^2}{4}}}$ is smallest when $e^{\frac{(x^2+y^2)^2}{4}}$ is biggest

e^m is an increasing function of m
make m smallest to make e^m smallest
.. m biggest to make e^m biggest.



$$m = \frac{1}{e^{\text{smallest}}} = \frac{1}{e^{\left(\frac{1}{4}\right)^2}} = \frac{1}{e^{1/16}}$$

$$M = \frac{1}{e^{\text{biggest}}} = \frac{1}{e^0} = 1$$

$$r = \sqrt{x^2+y^2} = \frac{1}{2} \\ r^2 = x^2+y^2 = \frac{1}{4} \\ (r^2)^2 = (x^2+y^2)^2 = \frac{1}{16} = \max \text{ of } (x^2+y^2)^2$$

$$\frac{1}{16} A \leq \int_0^{\frac{1}{2}} \int_0^{\sqrt{\frac{1}{4}-x^2}} e^{-\frac{(x^2+y^2)^2}{4}} dy dx \leq A$$

$$A = \frac{1}{4} \left[\pi \left(\frac{1}{2} \right)^2 \right] = \frac{1}{4} \cdot \frac{1}{4} \pi = \frac{1}{16} \pi$$

$$\left(\frac{1}{16} \right) \left(\frac{1}{16} \pi \right) \leq \iint_Q f dA \leq 1 \left(\frac{1}{16} \pi \right)$$

$$\boxed{\frac{1}{256} \leq \iint_Q f dA \leq \frac{\pi}{16}}, \text{ where } f = f(x, y) = e^{-\frac{(x^2+y^2)^2}{4}}$$