

Section 15.2 Double/Iterated Integrals

$$R = [a, b] \times [c, d].$$

S 15.2 #s1, 4, 7, 10, 15, 19, 27, 31, 38 from Handout.

Partial Integration with respect to y :
$$\int_c^d f(x, y) dy$$
 means hold x fixed and integrate with respect to y .

$$A(x) = \int_c^d f(x, y) dy$$
 is a function of x .
If we now integrate the function A with respect to x from $x = a$ to $x = b$, we get

$$\int_a^b A(x) dx = \int_a^b \left[\int_c^d f(x, y) dy \right] dx \text{ is called an iterated integral.}$$

Written in this fashion, it says to integrate w.r.t. y and then integrate w.r.t. x .**EXAMPLE 1** Evaluate the iterated integrals.

(a)
$$\int_0^3 \int_1^2 x^2 y dy dx$$

(b)
$$\int_1^2 \int_0^3 x^2 y dx dy$$

Before finishing this example, notice that Fubini says they better both come out the same.

FUBINI'S THEOREM If f is continuous on the rectangle
 $R = \{(x, y) \mid a \leq x \leq b, c \leq y \leq d\}$, then

$$\iint_R f(x, y) dA = \int_a^b \int_c^d f(x, y) dy dx = \int_c^d \int_a^b f(x, y) dx dy$$

More generally, this is true if we assume that f is bounded on R , f is discontinuous only on a finite number of smooth curves, and the iterated integrals exist.

If you can factor f thus: $f(x, y) = g(x)h(y)$ then $\iint_R f(x, y) dA = \int_a^b g(x) dx \int_c^d h(y) dy$

Don't expect this situation to arise a whole lot, but when it does, it's very nice.

5. $\int_0^2 \int_0^{\pi/2} x \sin y dy dx$