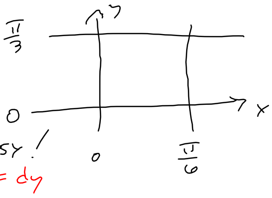
3–14 Calculate the iterated integral.

19. $\iint_R x\sin(x+y)\,dA, \quad R = [0, \pi/6] \times [0, \pi/3]$



$$= \int_0^{\pi/6}\int_0^{\pi/3} \sin(x+y)\,dy\,dx$$

HANDY MISCOPY TO MAKE IT O EASY !

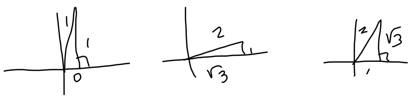
$\int \sin(u)\,du \quad u = x+y \quad du = dy$

$$= \int_0^{\pi/6}\Big[-\cos(x+y)\Big]_0^{\pi/3} dx = \int_0^{\pi/6}\Big[-\cos\left(x+\tfrac{\pi}{3}\right) - \left(-\cos(x+0)\right)\Big]dx$$

$$= \int_0^{\pi/6}\Big[-\cos\left(x+\tfrac{\pi}{3}\right)+\cos(x)\Big]dx = \Big[-\sin\left(x+\tfrac{\pi}{3}\right)+\sin(x)\Big]_0^{\pi/6}$$

$$= \left(-\sin\left(\tfrac{\pi}{2}\right)+\sin\left(\tfrac{\pi}{6}\right)\right) - \left(-\sin\left(\tfrac{\pi}{3}\right)+\sin(0)\right)$$

$$= \left(-1+\tfrac{1}{2}+\tfrac{\sqrt{3}}{2} = \tfrac{\sqrt{3}}{2}-\tfrac{1}{2}\right.$$



3–14 Calculate the iterated integral.

19. $\iint_R x\sin(x+y)\,dA, \quad R = [0, \pi/6] \times [0, \pi/3]$

$\sin(u+v) = \sin u\cos v + \sin v\cos u$

$$= \iint\left(x\sin x\cos y + x\sin y\cos x\right)dA$$

$$= \int_0^{\pi/6}\int_0^{\pi/3}\left(x\sin x\cos y + x\sin y\cos x\right)dy\,dx \overset{\text{FUBINI}}{=}$$

$$= \int_0^{\pi/6} x\sin x\,dx\int_0^{\pi/3}\cos y\,dy + \int_0^{\pi/6} x\cos x\,dx\int_0^{\pi/3}\sin y\,dy$$

EVAL "dx" portion

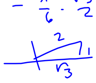
$u = x \Rightarrow du = dx$ | $u = x \quad du = dx$

$dv = \sin x\,dx \Rightarrow v = -\cos x$ | $dv = \cos x\,dx \quad v = \sin x$

$uv - \int v\,du =$ | $+ \quad uv - \int v\,du$

$-x\cos x\Big]_0^{\pi/6} - \int_0^{\pi/6} -\cos x\,dx$ | $x\sin x\Big]_0^{\pi/6} - \int_0^{\pi/6}\sin x\,dx$

$-\frac{\pi}{6}\cdot\frac{\sqrt{3}}{2} - 0 + \sin x\Big]_0^{\pi/6}$ | $\frac{\pi}{6}\cdot\frac{1}{2} - 0 - \Big[-\cos x\Big]_0^{\pi/6}$



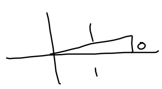
$= -\frac{\sqrt{3}\pi}{12} + \frac{1}{2}$ | $\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

This gives

$$\left(-\frac{\sqrt{3}\pi}{12}+\frac{1}{2}\right)\int_0^{\pi/3}\cos y\,dy + \left(\frac{\pi}{12}+\frac{\sqrt{3}}{2}-1\right)\int_0^{\pi/3}\sin y\,dy$$

$$= (\quad)\Big[\sin y\Big]_0^{\pi/3} + (\quad)\left(-\cos y\right)_0^{\pi/3}$$

$$= (\quad)\Big[\sin\tfrac{\pi}{3} - \sin 0\Big] + (\quad)\Big[-\cos\tfrac{\pi}{3} - (-\cos 0)\Big]$$



$$= \left(-\frac{\sqrt{3}\pi}{12}+\frac{1}{2}\right)\left[\frac{\sqrt{3}}{2}\right] + \left(\frac{\pi}{12}+\frac{\sqrt{3}}{2}-1\right)\left[-\frac{1}{2}+1\right] \quad \left[\tfrac{1}{2}\right]$$

$\frac{4\pi}{24} = \frac{\pi}{6}$

$$= -\frac{3\pi}{24} + \frac{\sqrt{3}}{4} + \frac{\pi}{24} + \frac{\sqrt{3}}{4} - \frac{1}{2}$$

$$= -\frac{2\pi}{24} + \frac{\pi}{12} + \cancel{\frac{\sqrt{3}}{4}} - \cancel{\frac{\sqrt{3}}{4}} + \frac{\sqrt{3}}{2} - \frac{1}{2} = \left(\frac{\pi}{} + \frac{\sqrt{3}}{} - \frac{1}{}\right.$$

27. Find the volume of the solid lying under the elliptic paraboloid $x^2/4 + y^2/9 + z = 1$ and above the rectangle $R = [-1, 1] \times [-2, 2]$.

NOTE: Evenness means $\int_{-a}^{a} \text{even} = 2\int_0^a$

$z = 1 - x^2/4 - y^2/9$

$$\int_{-1}^{1}\int_{-2}^{2}\left[1 - \frac{x^2}{4} - \frac{y^2}{9}\right]dy\,dx = \int_{-1}^{1}\left[y - \frac{x^2 y}{4} - \frac{y^3}{27}\right]_{-2}^{2}$$

$$= \int_{-1}^{1}\left[\left(2 - \frac{x^2}{2} - \frac{8}{27}\right) - \left(-2 + \frac{x^2}{2} + \frac{8}{27}\right)\right]dx$$

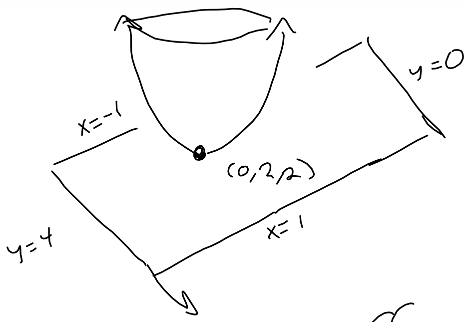
$$= \int_{-1}^{1}\left[4 - x^2 - \frac{16}{27}\right]dx = \left[4x - \frac{x^3}{3} - \frac{16}{27}x\right]_{-1}^{1}$$

$$= 4 - \frac{1}{3} - \frac{16}{27} - \left[-4 - (-1) - \left(-\frac{16}{27}\right)\right]$$

$$= 8 - \frac{2}{3} - \frac{32}{27} = \frac{216 - 18 - 32}{27} = \frac{216 - 50}{27} = \boxed{\frac{166}{27}}$$

$8 \overset{5}{(27)} = 216$

31. Find the volume of the solid enclosed by the paraboloid $z = 2 + x^2 + (y-2)^2$ and the planes $z = 1$, $x = 1$, $x = -1$, $y = 0$, and $y = 4$.



$dA = dx\,dy$ or $dy\,dx$

$$\iint \text{upper} - \text{lower} = \iint z - 1 = \iint \left(1 + x^2 (y-2)^2\right) dA$$

$$= \int_{-1}^{1}\int_{0}^{4} \left((1 + x^2 (y-2)^2\right) dy\,dx = \int_{-}^{1} \left[y + x^2 y - \frac{(y-2)^3}{3}\right]_0^4$$

$$= \int_{-1}^{1} \left(4 + 4x^2 - \frac{8}{3}\right) dx = \int_{-1}^{1} \left(\frac{4}{3} + 4x^2\right) dx = 2\int_0^1 \left(\frac{4}{3} + 4x^2\right) dx$$

$$= 2\left[\frac{4}{3}x + \frac{4}{3}x^3\right]_0^1 = 2\left[\frac{4}{3} + \frac{4}{3}\right] = 2\left[\frac{8}{3}\right] = \boxed{\frac{16}{3} = \text{Volume}}$$