

Shorthand:

There is, there exists

For all, for each, for every

$x$  is in the set  $S$ ,  $x$  is an element of  $S$

$A$  implies  $B$

$A$  implies  $B$  and  $B$  implies  $A$ , if and only if,  $A$  is necessary and sufficient to  $B$

Recall the definition of limit from Calculus I:

**1 Definition** Let  $f$  be a function of two variables whose domain  $D$  includes points arbitrarily close to  $(a, b)$ . Then we say that the **limit of  $f(x, y)$  as  $(x, y)$  approaches  $(a, b)$  is  $L$**  and we write

$$\lim_{(x, y) \rightarrow (a, b)} f(x, y) = L$$

if for every number  $\varepsilon > 0$  there is a corresponding number  $\delta > 0$  such that

if  $(x, y) \in D$  and  $0 < \sqrt{(x - a)^2 + (y - b)^2} < \delta$  then  $|f(x, y) - L| < \varepsilon$

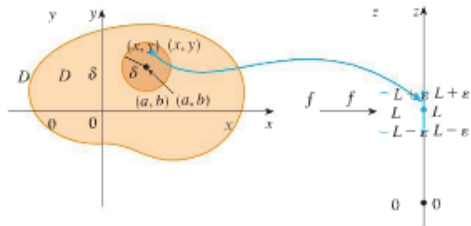


FIGURE 1

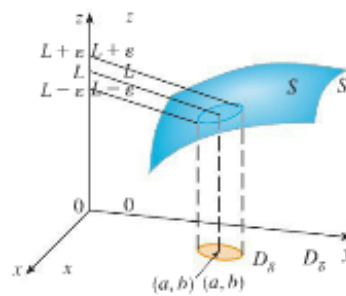


FIGURE 2

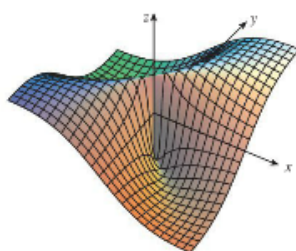
If  $f(x, y) \rightarrow L_1$  as  $(x, y) \rightarrow (a, b)$  along a path  $C_1$  and  $f(x, y) \rightarrow L_2$  as  $(x, y) \rightarrow (a, b)$  along a path  $C_2$ , where  $L_1 \neq L_2$ , then  $\lim_{(x,y) \rightarrow (a,b)} f(x, y)$  does not exist.

When the limit does NOT exist, sometimes the only practical way to show it is to be clever in how you make the approach to the limiting input value from a direction (or along a curve) where you get two different results, proving the limit doesn't exist.

Standard "trick."

**EXAMPLE 1** Show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$  does not exist.

! a rotating line on  
e 6 shows differ-  
gin from different



**FIGURE 6**

$$f(x, y) = \frac{xy}{x^2 + y^2}$$

Maybe y'all can help me grok the enrichment tool:

[https://www.cengage.com/math/discipline\\_content/  
stewartcalc7/2008/14\\_cengage\\_tec/publish/deployments/transcendentals\\_7e/  
7e\\_m12\\_6a.html](https://www.cengage.com/math/discipline_content/stewartcalc7/2008/14_cengage_tec/publish/deployments/transcendentals_7e/7e_m12_6a.html)



**EXAMPLE 4** Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2y}{x^2 + y^2}$  if it exists.

It DOES!!!

**5-22** Find the limit, if it exists, or show that the limit does not exist.

5.  $\lim_{(x,y) \rightarrow (1,2)} (5x^3 - x^2y^2)$

6.  $\lim_{(x,y) \rightarrow (1,-1)} e^{-xy} \cos(x+y)$

10.  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + \sin^2 y}{2x^2 + y^2}$

**29–38** Determine the set of points at which the function is continuous.

29.  $F(x, y) = \frac{\sin(xy)}{e^x - y^2}$

30.  $F(x, y) = \frac{x - y}{1 + x^2 + y^2}$

31.  $F(x, y) = \arctan(x + \sqrt{y})$

32.  $F(x, y) = e^{x^2y} + \sqrt{x + y^2}$

These are basically "Look for holes in the domain, and those will be where it's not continuous, generally speaking." 99.9% of the functions that have a formula you can write down are continuous on their entire domain.

**39–41** Use polar coordinates to find the limit. [If  $(r, \theta)$  are polar coordinates of the point  $(x, y)$  with  $r \geq 0$ , note that  $r \rightarrow 0^+$  as  $(x, y) \rightarrow (0, 0)$ .]

**39.**  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + y^3}{x^2 + y^2}$

**40.**  $\lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \ln(x^2 + y^2)$