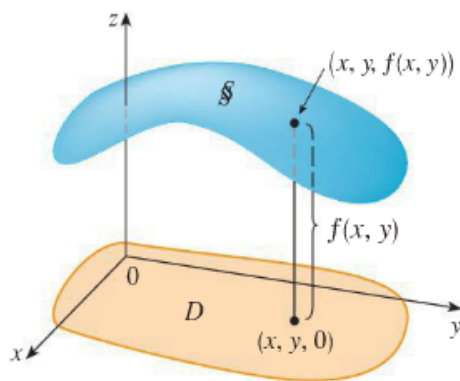


14.1 #s 13, 15, 17-19, 23, 24, 36, 41, 43, 45, 53, 54, 57, 58

**Definition** A function  $f$  of two variables is a rule that assigns to each ordered pair of real numbers  $(x, y)$  in a set  $D$  a unique real number denoted by  $f(x, y)$ . The set  $D$  is the **domain** of  $f$  and its **range** is the set of values that  $f$  takes on, that is,  $\{f(x, y) \mid (x, y) \in D\}$ .

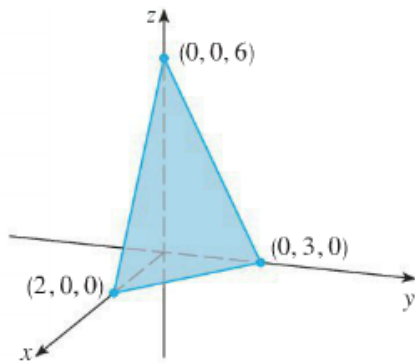


**Definition** If  $f$  is a function of two variables with domain  $D$ , then the **graph** of  $f$  is the set of all points  $(x, y, z)$  in  $\mathbb{R}^3$  such that  $z = f(x, y)$  and  $(x, y)$  is in  $D$ .



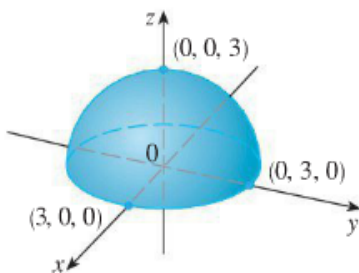
**FIGURE 5**

**EXAMPLE 5** Sketch the graph of the function  $f(x, y) = 6 - 3x - 2y$ .



**FIGURE 6**

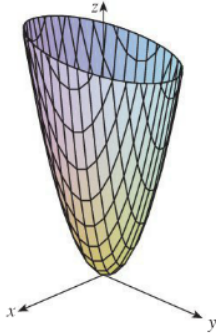
**EXAMPLE 6** Sketch the graph of  $g(x, y) = \sqrt{9 - x^2 - y^2}$ .



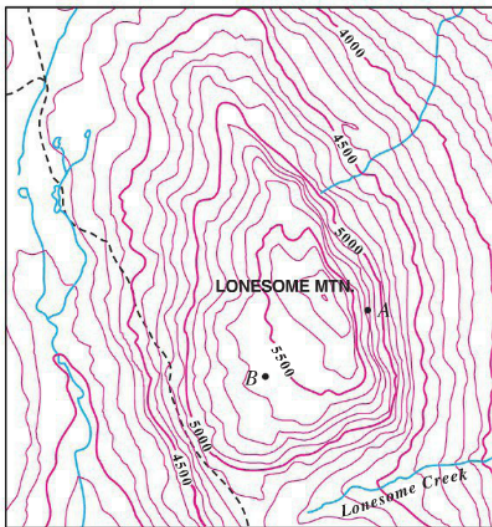
**FIGURE 7**

Graph of  $g(x, y) = \sqrt{9 - x^2 - y^2}$

**EXAMPLE 8** Find the domain and range and sketch the graph of  $h(x, y) = 4x^2 + y^2$ .



**Definition** The **level curves** of a function  $f$  of two variables are the curves with equations  $f(x, y) = k$ , where  $k$  is a constant (in the range of  $f$ ).



Topographic Maps!

FIGURE 12

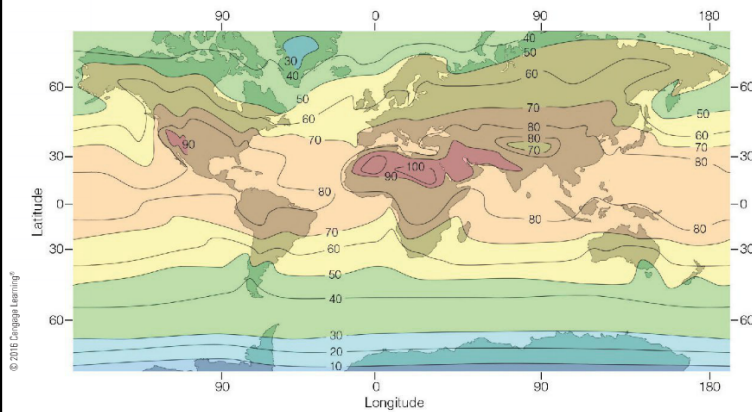


FIGURE 13 Average air temperature near sea level in July ( $^{\circ}\text{F}$ )

**EXAMPLE 12** Sketch some level curves of the function  $h(x, y) = 4x^2 + y^2 + 1$

This is our friend, the elliptical paraboloid, opening up, with vertex at  $(0, 0, 1)$ .

11-20 Find and sketch the domain of the function.

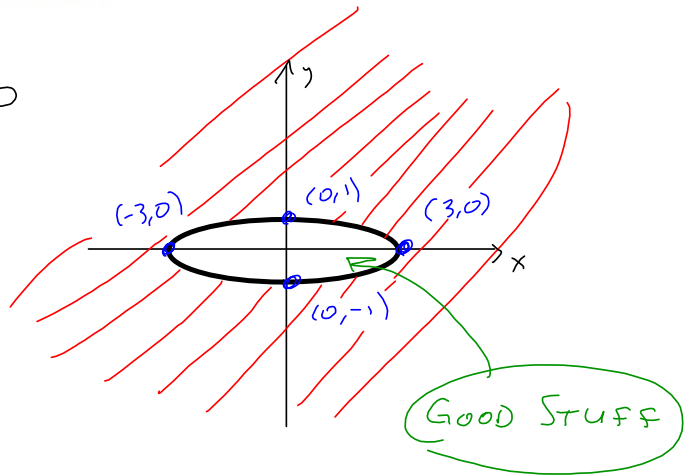
13.  $f(x, y) = \ln(9 - x^2 - 9y^2)$

Need  $9 - x^2 - 9y^2 > 0$

$x^2 + 9y^2 < 9$

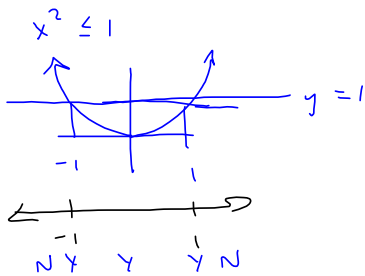
$\frac{x^2}{9} + y^2 < 1$

$\frac{x^2}{3^2} + \frac{y^2}{1^2}$



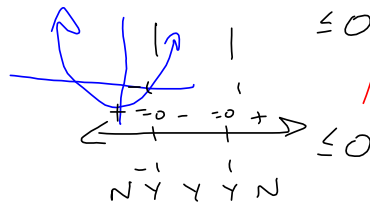
15.  $f(x, y) = \sqrt{1 - x^2} - \sqrt{1 - y^2}$

Need  $1 - x^2 \geq 0$  and  $1 - y^2 \geq 0$



$1 - x^2 \geq 0$   
 $(1-x)(1+x) \geq 0$   
 $-(x-1)(x+1) \geq 0$   
 $(x-1)(x+1) \leq 0$

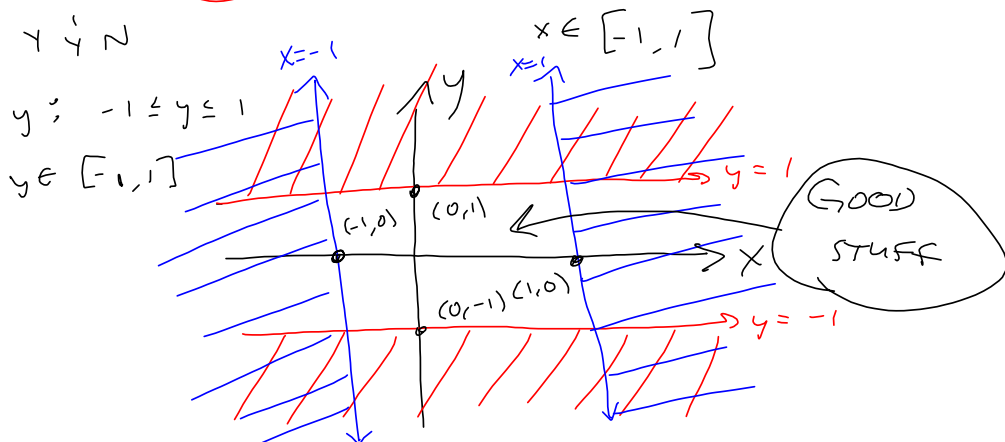
Test  $x=2$   
 $(2-1)(2+1) > 0$  (+)



AND  
 $1 - y^2 \geq 0 \Rightarrow$   
 $y^2 - 1 \leq 0$

$(y-1)(y+1) \leq 0$   
 + = 0 - = 0 +  $\leq 0$   
 N - + - N

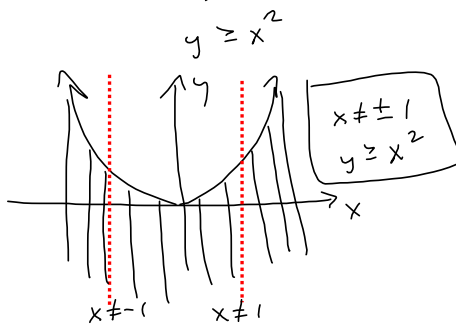
Find one sign.  
 Manage sign changes.



17.  $f(x, y) = \frac{\sqrt{y - x^2}}{1 - x^2}$

Scratch out the bad stuff. Keep the good stuff.

Need  $y - x^2 \geq 0$  and  $1 - x^2 \neq 0$



18.  $f(x, y) = \arcsin(x^2 + y^2 - 2)$

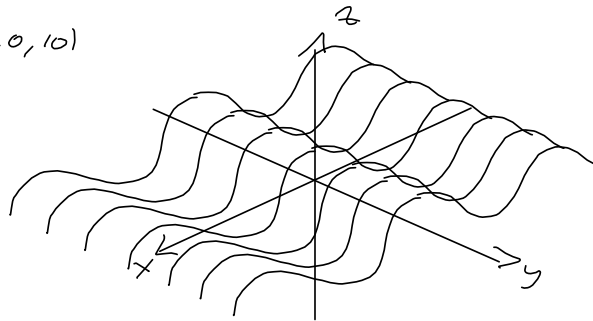
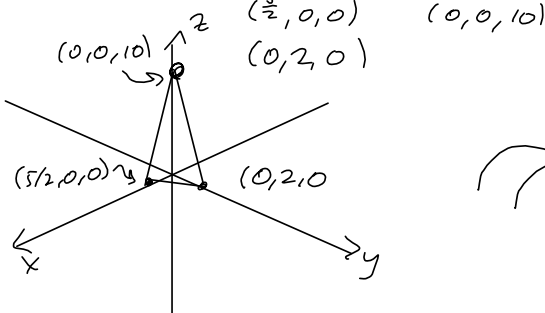
$$19. f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$$

**21-29** Sketch the graph of the function.

**23.**  $f(x, y) = 10 - 4x - 5y$

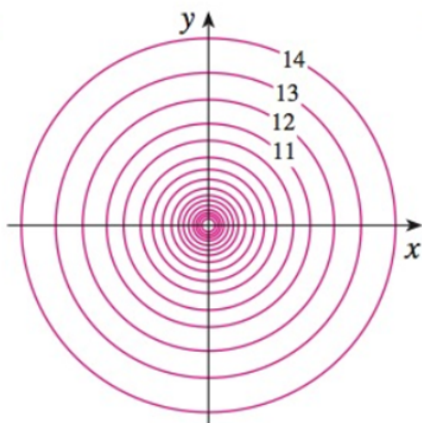
**24.**  $f(x, y) = \cos x \quad z = \cos x$

$z = 10 - 4x - 5y \Rightarrow 4x + 5y + z = 10$

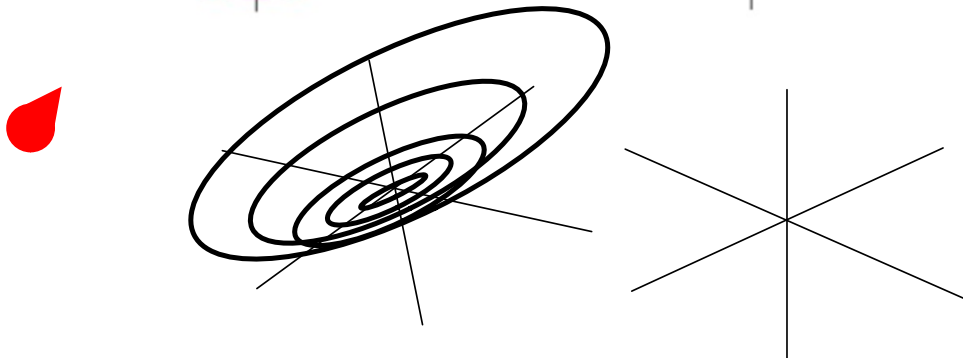
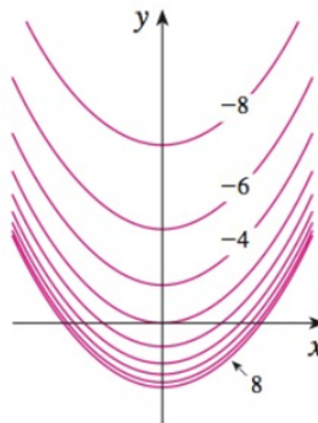


**35-38** A contour map of a function is shown. Use it to make a rough sketch of the graph of  $f$ . #35 NA

**35.**



**36.**



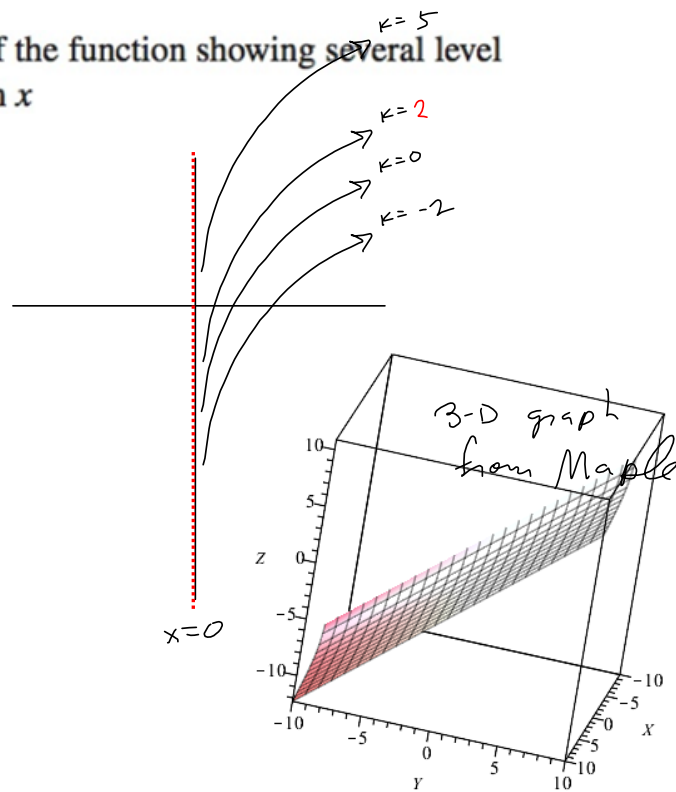


**39-46** Draw a contour map of the function showing several level curves. **41.**  $f(x, y) = y - \ln x$

$$y - \ln(x) = k$$

$$y = \ln(x) + k$$

$$y = \ln(x) + 2$$



**43.**  $f(x, y) = ye^x$

**51-54** Use a computer to graph the function using various domains and viewpoints. Get a printout of one that, in your opinion, gives a good view. If your software also produces level curves, then plot some contour lines of the same function and compare with the graph. #s 51-2 NA

**51.**  $f(x, y) = e^{-x^2} + e^{-2y^2}$     **52.**  $f(x, y) = (1 - 3x^2 + y^2)e^{1-x^2-y^2}$

$$\frac{1}{e^{x^2}} + \frac{1}{e^{2y^2}}$$

