Recall: Arc Length for $\bar{r}(t) = \langle x(t), y(t), z(t) \rangle$

from $t = \alpha$ to $t = \beta$

$$L = s = \int_\alpha^\beta \underbrace{\sqrt{x'^2 + y'^2 + z'^2}\, dt}_{ds}$$

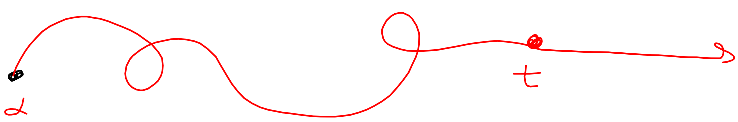
$$s = s(t) = \int_\alpha^t \sqrt{x'(u)^2 + y'(u)^2 + z'(u)^2}\, du = \int_\alpha^t \|\bar{r}'(u)\|\, du$$

$$\Longrightarrow \frac{ds}{dt} = s'(t) = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\ \boxed{= \|\bar{r}'(t)\| = \frac{ds}{dt}}$$

$\boxed{D}$ $\kappa = \left\|\frac{d\bar{T}}{ds}\right\|$. If $t = t(s)$ is function of s, then chain rule:

$$\frac{d\bar{T}}{dt} = \frac{d\bar{T}}{ds} \cdot \frac{ds}{dt}, \text{ so}$$

$$\left\|\frac{d\bar{T}}{ds}\right\| = \left\|\frac{\frac{d\bar{T}}{dt}}{\frac{ds}{dt}}\right\| = \frac{\|\bar{T}'(t)\|}{\|\bar{r}'\|}$$



T10 $\kappa(t) = \dfrac{\|\vec{r}\,' \times \vec{r}\,''\|}{\|\vec{r}\,'\|^3} = \|\vec{r}\,'(t)\|$, etc.

Proof $\vec{T} = \dfrac{\vec{r}\,'}{\|\vec{r}\,'\|} \Longrightarrow \vec{r}\,' = \|\vec{r}\,'\|\,\vec{T} =$

$$\Longrightarrow \vec{r}\,'' = (\|\vec{r}\,'\|)'\vec{T} + \|\vec{r}\,'\|\,\vec{T}\,'$$
$$= \left(\frac{ds}{dt}\right)'\vec{T} + \frac{ds}{dt}\vec{T}\,'$$
$$= \frac{d^2s}{dt^2}\vec{T} + \frac{ds}{dt}\vec{T}\,'$$

Consider $\vec{r}\,' \times \vec{r}\,''$

$$= \vec{r}\,' \times \left(\frac{d^2s}{dt^2}\vec{T}\right) + \vec{r}\,' \times \frac{ds}{dt}\vec{T}\,'$$

$\vec{T} = \dfrac{\vec{r}\,'}{\|\vec{r}\,'\|} \Longrightarrow$

$$= \left(\frac{d^2s}{dt^2}\right)(\vec{r}\,' \times \vec{T}) + \left(\frac{ds}{dt}\right)\vec{r}\,' \times \vec{T}\,'$$

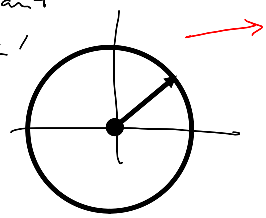
$\vec{r}\,' = \|\vec{r}\,'\|\,\vec{T}$

$$= \underbrace{\boxed{\frac{d^2s}{dt^2}\left(\|\vec{r}\,'\|\,\vec{T} \times \vec{T}\right)}}_{=0!} + \left(\frac{ds}{dt}\right)(\vec{r}\,' \times \vec{T}\,')$$

$\|\vec{T} \times \vec{T}\|$
$= \|\vec{T}\|^2 \sin\theta = 0$

$$= \left(\frac{ds}{dt}\right)(\vec{r}\,' \times \vec{T}\,')$$

$\|\vec{r}\| = \text{constant}$
$\Longrightarrow \vec{r} \perp \vec{r}\,'$



$$= \boxed{\|\vec{r}\,'\|}\left(\|\vec{r}\,'\|\,\vec{T} \times \vec{T}\,'\right)$$
$$= \|\vec{r}\,'\|^2\,\|\vec{T}\|\,\|\vec{T}\,'\|\sin\theta \qquad \& \quad \vec{T} \perp \vec{T}\,'$$
$$= \left(\frac{ds}{dt}\right)^2 \|\vec{T} \times \vec{T}\,'\| = \|\vec{r}\,' \times \vec{r}\,''\|$$
$$\Longrightarrow \left(\frac{ds}{dt}\right)^2 \|\vec{T}\,'\| = \|\vec{r}\,' \times \vec{r}\,''\|$$
$$\Longrightarrow \|\vec{T}\,'\| = \frac{\|\vec{r}\,' \times \vec{r}\,''\|}{\left(\frac{ds}{dt}\right)^2} = \frac{\|\vec{r}\,' \times \vec{r}\,''\|}{\|\vec{r}\,'\|^2}$$
$$\Longrightarrow \kappa = \left|\frac{d\vec{T}}{ds}\right| = \frac{\frac{d\vec{T}}{dt}}{\frac{ds}{dt}} = \frac{\|\vec{T}\,'\|}{\|\vec{r}\,'\|} = \frac{\frac{\|\vec{r}\,' \times \vec{r}\,''\|}{\|\vec{r}\,'\|^2}}{\|\vec{r}\,'\|}$$

$$\boxed{\kappa = \frac{\|\vec{r}\,' \times \vec{r}\,''\|}{\|\vec{r}\,'\|^3}}$$