1. The figure shows a curve C given by a vector function $\mathbf{r}(t)$.

(a) Draw the vectors $\mathbf{r}(4.5) - \mathbf{r}(4)$ and $\mathbf{r}(4.2) - \mathbf{r}(4)$.

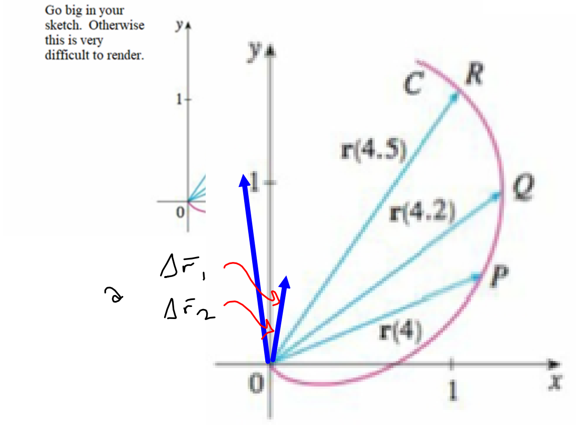
(b) Draw the vectors

$$\frac{\mathbf{r}(4.5) - \mathbf{r}(4)}{0.5} \quad \text{and} \quad \frac{\mathbf{r}(4.2) - \mathbf{r}(4)}{0.2}$$

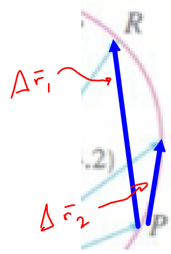
(c) Write expressions for $\mathbf{r}'(4)$ and the unit tangent vector $\mathbf{T}(4)$.

(d) Draw the vector $\mathbf{T}(4)$.

Go big in your sketch. Otherwise this is very difficult to render.



(a)



$\Delta\bar{r}_1 = \bar{r}(4.5) - \bar{r}(4)$

$\Delta\bar{r}_2 = \bar{r}(4.2) - \bar{r}(4)$

(b) $\dfrac{\bar{r}(4.5) - \bar{r}(4)}{0.5} = \dfrac{\sim}{\frac{1}{2}} = 2\cdot \sim$

$= 2\left(\bar{r}(4.5) - \bar{r}(4)\right)$

$= \dfrac{\bar{r}(4.2) - \bar{r}(4)}{0.2} = \dfrac{\sim}{\frac{1}{5}} = 5\cdot \sim$

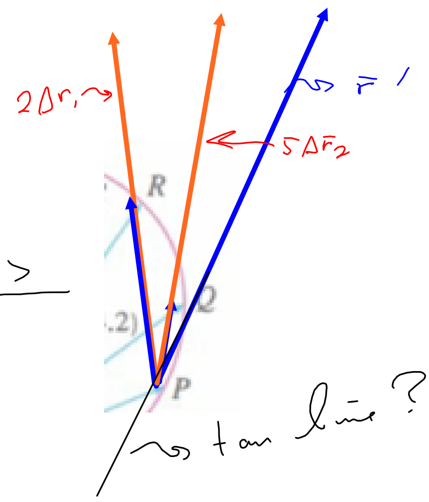
$= 5\left(\bar{r}(4.2) - \bar{r}(4)\right)$

(c) $\bar{T}(t) = \dfrac{\bar{r}'(t)}{\|\bar{r}'(t)\|} = \dfrac{\langle x'(t), y'(t), z'(t)\rangle}{\sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}}$

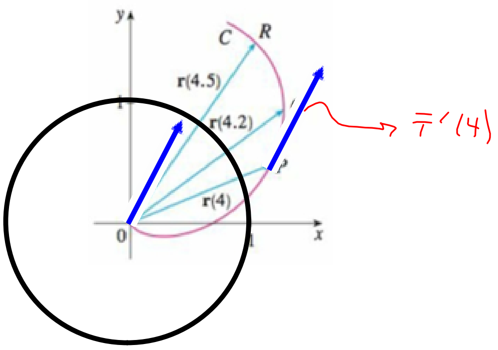
$\bar{r}'(t) = \lim\limits_{h\to 0} \dfrac{\bar{r}(4+h) - \bar{r}(4)}{h} =$

$= \lim\limits_{h\to 0} \dfrac{\langle x(4+h), y(4+h), z(4+h)\rangle - \langle x(4), y(4), z(4)\rangle}{h}$

$\bar{T}(4) = \dfrac{\langle x'(4), y'(4), z'(4)\rangle}{\sqrt{x'(4)^2 + y'(4)^2 + z'(4)^2}}$



(d)



Feb 3-12:53 PM

3 THEOREM Suppose **u** and **v** are differentiable vector functions, c is a scalar, and f is a real-valued function. Then

1. $\frac{d}{dt}[\mathbf{u}(t) + \mathbf{v}(t)] = \mathbf{u}'(t) + \mathbf{v}'(t)$
2. $\frac{d}{dt}[c\mathbf{u}(t)] = c\mathbf{u}'(t)$
3. $\boxed{\frac{d}{dt}[f(t)\mathbf{u}(t)] = f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t)}$
4. $\frac{d}{dt}[\mathbf{u}(t) \cdot \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$
5. $\boxed{\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)}$
6. $\frac{d}{dt}[\mathbf{u}(f(t))] = f'(t)\mathbf{u}'(f(t))$ (Chain Rule)

2-D: 1 $\bar{u}(t) = \langle x_1(t), y_1(t) \rangle$

$\bar{v}(t) = \langle x_2(t), y_2(t) \rangle$

$\frac{d}{dt}\bar{v}(t) = \frac{d}{dt}\langle x_1(t) + x_2(t), y_1(t) + y_2(t) \rangle$

$= \langle \frac{d}{dt}(x_1(t) + x_2(t)), \frac{d}{dt}(y_1(t) + y_2(t)) \rangle$

$= \langle \frac{dx_1(t)}{dt} + \frac{dx_2(t)}{dt}, \frac{dy_1(t)}{dt} + \frac{dy_2(t)}{dt} \rangle$

$= \ldots \quad \bar{u}'(t) + \bar{v}'(t)$.

3 $\frac{d}{dt}(f(t)\,\bar{u}(t)) = \frac{d}{dt} f(t) \langle x_1(t), y_1(t) \rangle$

$= \frac{d}{dt}\langle f(t)x_1(t), f(t)y_1(t) \rangle$

$= \langle \frac{d}{dt}(f(t)x_1(t))', \frac{d}{dt}(f(t)y_1(t)) \rangle$

$(fg)' = f'g + fg'$

You need to supply more than I'm giving

$= f'(t)\bar{u}(t) + f(t)\bar{u}'(t)$

$\frac{d}{dt}(\bar{u} \times \bar{v}) =$

$\frac{d\bar{u}}{dt} \times \bar{v}(t) + \bar{u}(t) \times \bar{v}'(t)$

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3 THEOREM Suppose $\mathbf{u}$ and $\mathbf{v}$ are differentiable vector functions, c is a scalar, and f is a real-valued function. Then

1. $\frac{d}{dt}[\mathbf{u}(t) + \mathbf{v}(t)] = \mathbf{u}'(t) + \mathbf{v}'(t)$
2. $\frac{d}{dt}[c\mathbf{u}(t)] = c\mathbf{u}'(t)$
3. $\frac{d}{dt}[f(t)\mathbf{u}(t)] = f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t)$
4. $\frac{d}{dt}[\mathbf{u}(t) \cdot \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$
5. $\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$
6. $\frac{d}{dt}[\mathbf{u}(f(t))] = f'(t)\mathbf{u}'(f(t))$ (Chain Rule)

Proof of (5)

$\bar{u} = \langle x_1, y_1, z_1 \rangle, \ \langle x_2, y_2, z_2 \rangle = \bar{v}$

$f(t) = f$

$\frac{d}{dt}\left[\bar{u}(t) \times \bar{v}(t)\right]$

$x_1, \ y_1, \ z_1, \ x_1, \ y_1$
$x_2, \ y_2, \ z_2, \ x_2, \ y_2$

$= \frac{d}{dt}\left[\langle y_1 z_2 - y_2 z_1, \ x_2 z_1 - x_1 z_2, \ x_1 y_2 - x_2 y_1 \rangle\right]$

$= \langle \frac{d}{dt}(y_1 z_2) - \frac{d}{dt}(y_2 z_1), \ \frac{d}{dt}(x_2 z_1) - \frac{d}{dt}(x_1 z_2), \ \frac{d}{dt}(x_1 y_2) - \frac{d}{dt}(x_2 y_1) \rangle$

$= \langle \frac{dy_1}{dt} z_2 + y_1 \frac{dz_2}{dt} - \left[\frac{dy_2}{dt} z_1 + y_2 \frac{dz_1}{dt}\right],$

$= \ldots = \boxed{\bar{u}'(t)\bar{v}(t) \times \bar{u}(t)\bar{v}'(t)}$ end up

start here & see if you can meet what you've got, working backwards, toward

$\frac{d}{dt}\left[\bar{u}(t) \times \bar{v}(t)\right]$

9–16 Find the derivative of the vector function.

9. $\mathbf{r}(t) = \langle t \sin t, t^2, t \cos 2t \rangle$

13. $\mathbf{r}(t) = e^{t^2}\mathbf{i} - \mathbf{j} + \ln(1 + 3t)\,\mathbf{k}$ $= \langle e^{t^2}, -1, \ln(3t+1) \rangle$

13. $\mathbf{r}(t) = e^{t^2}\mathbf{i} - \mathbf{j} + \ln(1 + 3t)\,\mathbf{k}$

16. $\mathbf{r}(t) = t\,\mathbf{a} \times (\mathbf{b} + t\,\mathbf{c})$

#13

$\Rightarrow \boxed{\bar{r}\,'(t) = \langle 2te^{t^2}, 0, \frac{3}{3t+1} \rangle}$

16

$t\bar{a} \times (\bar{b} + t\bar{c}) = \bar{r}(t) \Rightarrow$

$\bar{r}\,'(t) = (1\bar{a} + t\bar{a}\,') \times (\bar{b} + t\bar{c}) + t\bar{a} \times (\bar{b}\,' + 1\bar{c} + t\bar{c}\,')$

Depends on book answer, frankly, but this seems legit

$\bar{a} \times \bar{b} + \bar{a} \times t\bar{c} + t\bar{a}\,' \times \bar{b} + t\bar{a}\,' \times (t\bar{c})$

$+ t\bar{a} \times \bar{b} + t\bar{a} \times (t\bar{c}\,')$

$(\bar{a} \times \bar{b})' = \bar{a}\,' \times \bar{b} + \bar{a} \times \bar{b}\,'$

17–20 Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t.

19. $\mathbf{r}(t) = \cos t\,\mathbf{i} + 3t\,\mathbf{j} + 2\sin 2t\,\mathbf{k}, \quad t = 0$

20. $\mathbf{r}(t) = 2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j} + \tan t\,\mathbf{k}, \quad t = \pi/4$

19) $\bar{r}(t) = \langle \cos(t), 3t, 2\sin(2t) \rangle \Longrightarrow$

$$\bar{T}(t) = \frac{\bar{r}'(t)}{\|\bar{r}(t)\|} = \frac{\langle -\sin(t), 3, 4\cos(2t) \rangle}{\sqrt{\sin^2(t) + 3^2 + (4\cos(2t))^2}}$$

$$\Longrightarrow \bar{T}(0) = \frac{\langle 0, 3, 4 \rangle}{\sqrt{0^2 + 3^2 + 4^2}} = \frac{1}{\sqrt{25}} \langle 0, 3, 4 \rangle \boxed{= \frac{1}{5} \langle 0, 3, 4 \rangle = \bar{T}(0)}$$

21. If $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$, find $\mathbf{r}'(t)$, $\mathbf{T}(1)$, $\mathbf{r}''(t)$, and $\mathbf{r}'(t) \times \mathbf{r}''(t)$.

22. If $\mathbf{r}(t) = \langle e^{2t}, e^{-2t}, te^{2t} \rangle$, find $\mathbf{T}(0)$, $\mathbf{r}''(0)$, and $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$.

23–26 Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

23. $x = 1 + 2\sqrt{t}$, $y = t^3 - t$, $z = t^3 + t$; $(3, 0, 2)$

24. $x = e^t$, $y = te^t$, $z = te^{t^2}$; $(1, 0, 0)$

25. $x = e^{-t} \cos t$, $y = e^{-t} \sin t$, $z = e^{-t}$; $(1, 0, 1)$

23 $\bar{r}(t) = \langle x(t), y(t), z(t) \rangle = \langle 2t^{\frac{1}{2}} + 1, t^3 - t, t^3 + t \rangle$

a $\bar{r}_0 = \langle 3, 0, 2 \rangle$ $2\sqrt{t} + 1 = 3$ ✓ $t = 1$ ✓

$1^3 - 1 = 0$ ✓

$1^3 + 1 = 2$ ✓

$\bar{r}'(t) = \langle t^{-\frac{1}{2}}, 3t^2 - 1, 3t^2 + 1 \rangle$

$\bar{r}'(1) = \langle 1, 3(1)^2 - 1, 3(1)^2 + 1 \rangle = \langle 1, 2, 4 \rangle = \bar{r}'(1) =$

the direction vector for our line!

$\bar{l}(t) = \bar{r}_0 + t \langle 1, 2, 4 \rangle = \langle 3, 0, 2 \rangle + t \langle 1, 2, 4 \rangle$

$= \langle t+3, 2t, 4t+2 \rangle$

Parametric eq'ns to make everybody happy:

$x = t + 3$, $y = 2t$, $z = 4t + 2$

27–29 Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point. Illustrate by graphing both the curve and the tangent line on a common screen.

27. $x = t,\ y = e^{-t},\ z = 2t - t^2;\quad (0, 1, 0)$

29. $x = t\cos t,\ y = t,\ z = t\sin t;\quad (-\pi, \pi, 0)$

27 $t = 0$ hits $(0,1,0)$ $\longrightarrow$ $\bar{r}_0 = \langle 0,1,0\rangle$ is "initial point"

$x = 0,\ y = e^{-0} = 1,\ z = 2(0) - 0^2 = 0$

$\bar{r}(t) = \langle t, e^{-t}, 2t - t^2\rangle \Longrightarrow$

$\bar{r}'(t) = \langle 1, -e^{-t}, 2 - 2t\rangle \Longrightarrow$

$\bar{r}'(0) = \langle 1, -1, 2\rangle$ is my direction vector

$\bar{L}(t) = \langle 0,1,0\rangle + t\langle 1,-1,2\rangle$

Link to Maple Transcript

Feb 3-9:28 PM