13.1 Vector Functions and Space Curves

In general, a function is a rule that assigns to each element in the domain an element in the range.

A **vector-valued function**, or **vector function**, is simply a function whose domain is a set of real numbers and whose range is a set of vectors.

We are most interested in vector functions **r** whose values are three-dimensional vectors.

This means that for every number *t* in the domain of **r** there is a unique vector in V_3 denoted by **r**(*t*).

$\vec{r}(t) = \langle t, \sin(t), \cos(t) \rangle$

Mapping from $\mathbb{R} \rightarrow \mathbb{R}^3$

$\vec{r}(0) = \langle 0, 0, 1 \rangle$

If $f(t)$, $g(t)$, and $h(t)$ are the components of the vector $\mathbf{r}(t)$, then f, g, and h are real-valued functions called the **component functions** of $\mathbf{r}$ and we can write

$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

We use the letter t to denote the independent variable because it represents time in most applications of vector functions.

Example

If $\mathbf{r}(t) = \langle t^3, \ln(3 - t), \sqrt{t} \rangle$

then the component functions are

$$f(t) = t^3 \qquad g(t) = \ln(3 - t) \qquad h(t) = \sqrt{t}$$

By our usual convention, the domain of **r** consists of all values of t for which the expression for $\mathbf{r}(t)$ is defined.

The expressions t^3, $\ln(3 - t)$, and $\sqrt{t}$ are all defined when $3 - t > 0$ and $t \geq 0$.

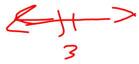
Therefore the domain of **r** is the interval $[0, 3)$.

t^3 : $\mathcal{D} = \mathbb{R}$

$\ln(3-t)$: Need $3 - t > 0$

$3 > t$

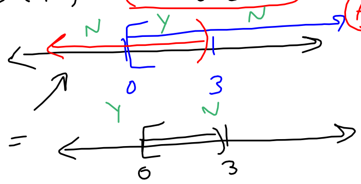
$t < 3$



$\mathcal{D} = (-\infty, 3)$

$\sqrt{t}$: Need $t \geq 0$ $\mathcal{D} = [0, \infty)$

$\mathcal{D}(\vec{r})$ = Intersection : AND



$\mathcal{D} = [0, 3)$

$\boxed{1}$ If $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, then

$$\lim_{t \to a} \mathbf{r}(t) = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$$

provided the limits of the component functions exist.

Nothin' new.
Just 3 times the work

A vector function **r** is **continuous at *a*** if

$$\lim_{t \to a} \mathbf{r}(t) = \mathbf{r}(a)$$

Nothin' new.
Just 3 times the work

In view of Definition 1, we see that **r** is continuous at *a* if and only if its component functions *f*, *g*, and *h* are continuous at *a*.

There is a close connection between continuous vector functions and space curves.

Suppose that f, g, and h are continuous real-valued functions on an interval I.

Then the set C of all points (x, y, z) in space, where

$$\boxed{2} \qquad x = f(t) \qquad y = g(t) \qquad z = h(t)$$

and t varies throughout the interval I, is called a **space curve**.

The equations in $\boxed{2}$ are called **parametric equations of *C*** and t is called a **parameter**.

We can think of C as being traced out by a moving particle whose position at time t is $(f(t), g(t), h(t))$.

"spacecurve" command, in Maple

If we now consider the vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$, then $\mathbf{r}(t)$ is the position vector of the point $P(f(t), g(t), h(t))$ on C.

Thus any continuous vector function $\mathbf{r}$ defines a space curve C that is traced out by the tip of the moving vector $\mathbf{r}(t)$, as shown in Figure 1.

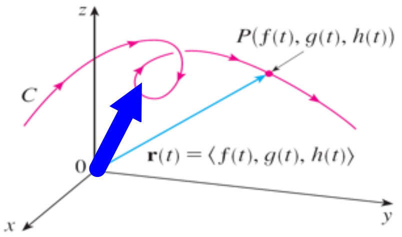


Figure 1

C is traced out by the tip of a moving position vector $\mathbf{r}(t)$.